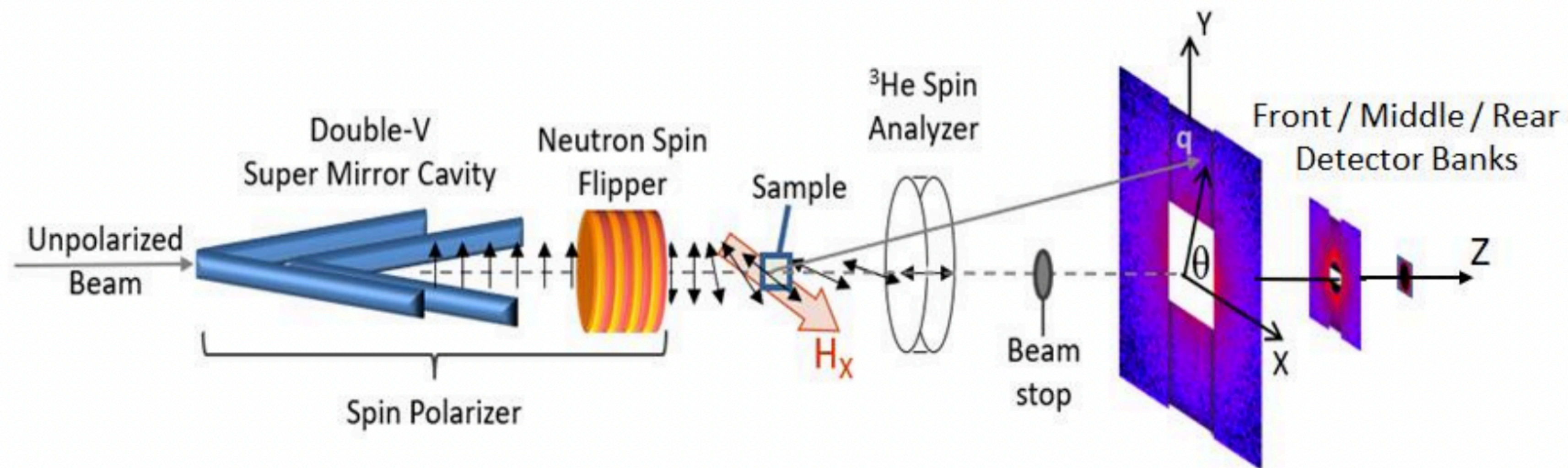


Small-Angle Neutron Scattering

And Polarization Analysis Of Magnetic Materials

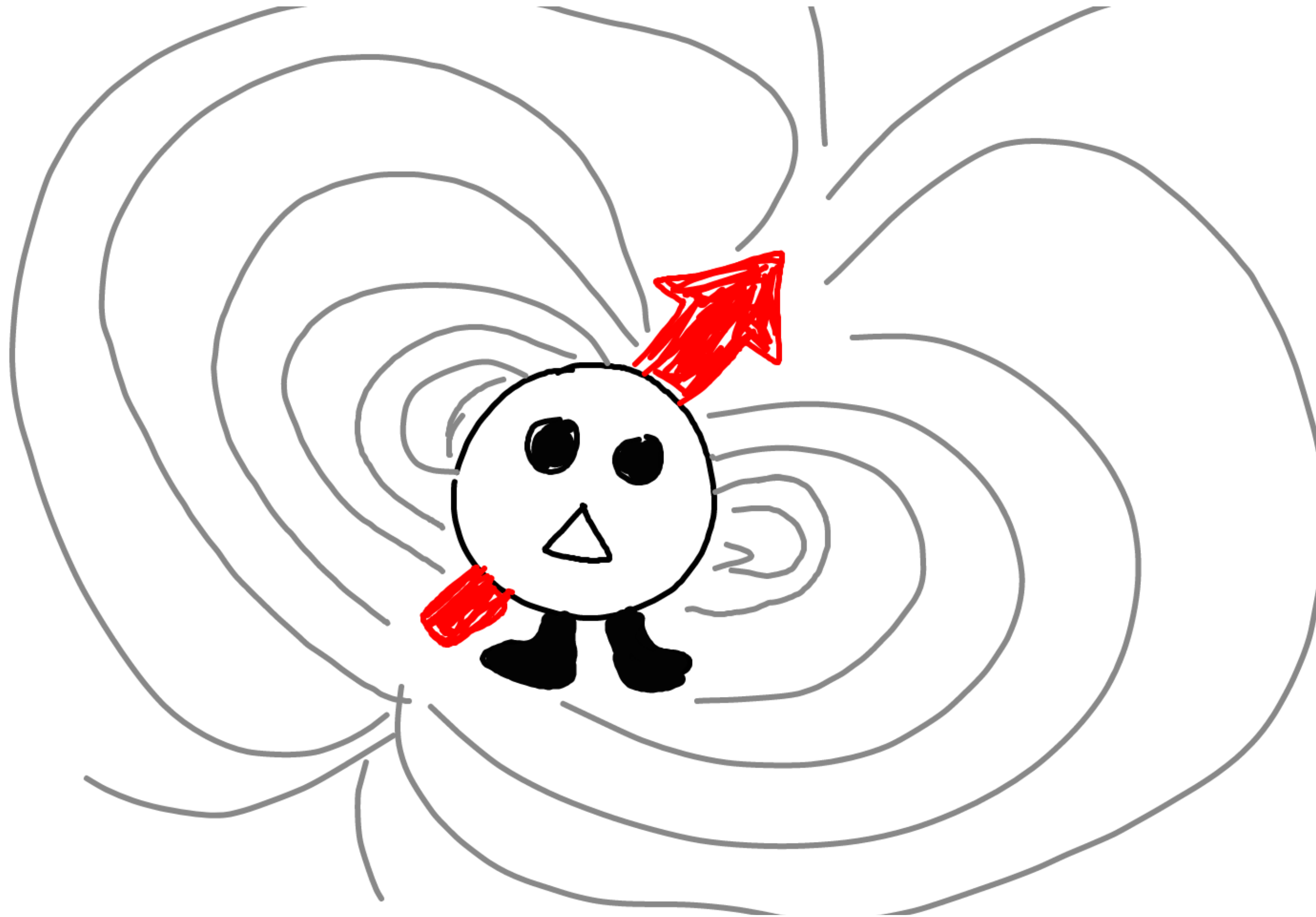


NCNR Summer School 2026

Goal of this talk

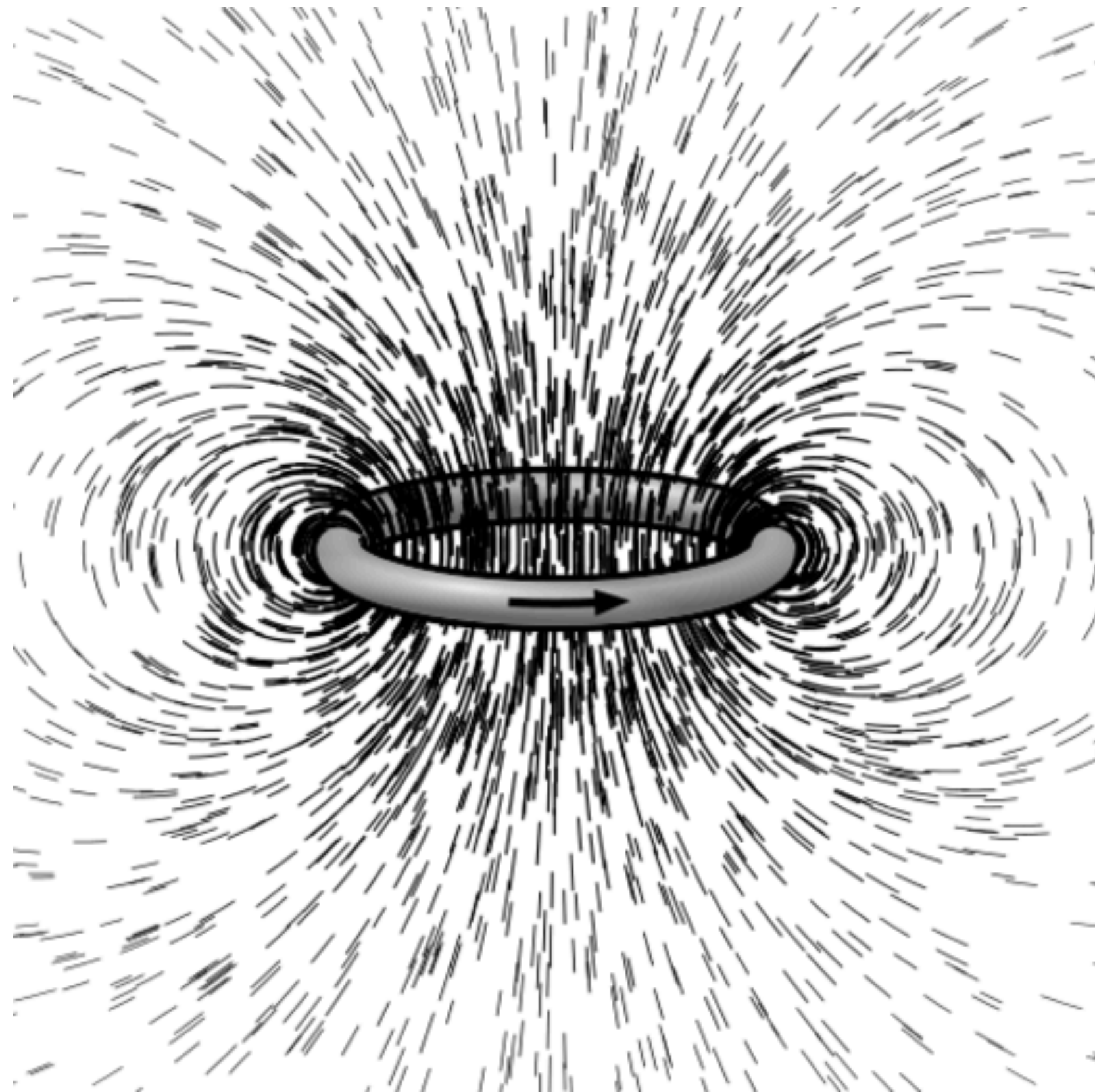
- What is Small-Angle Neutron Scattering (SANS) and how can we use it with polarized neutrons to gain insights into magnetic properties of materials?

The neutron properties are well tuned to study materials

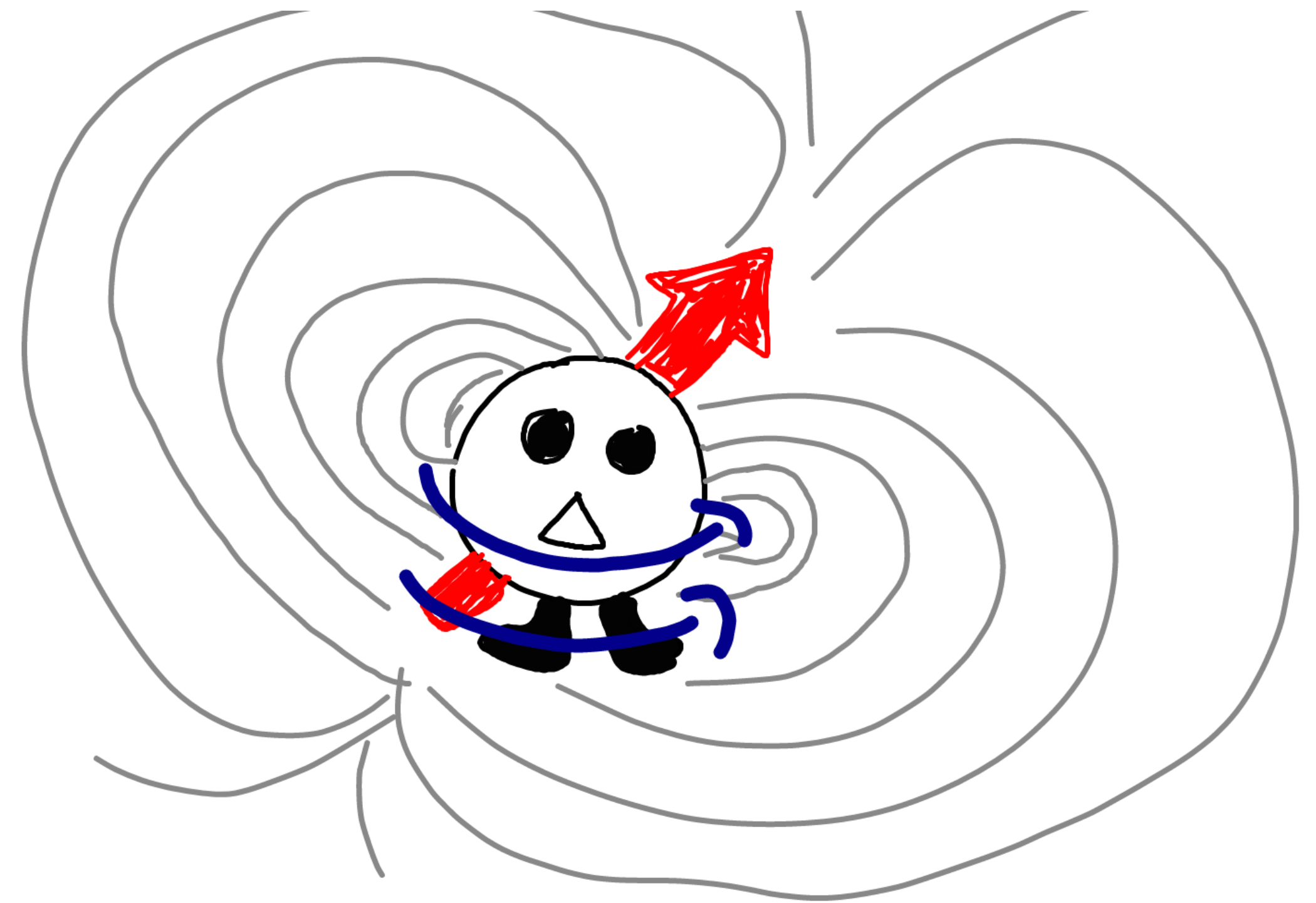


- Lifetime ~ 15 min
- No charge
- Mass = 1.67492×10^{-27} kg
- Spin = $1/2$

As a crude approximation, a spin can be seen as an intrinsic angular momentum (“spinning”) that produces a magnetization



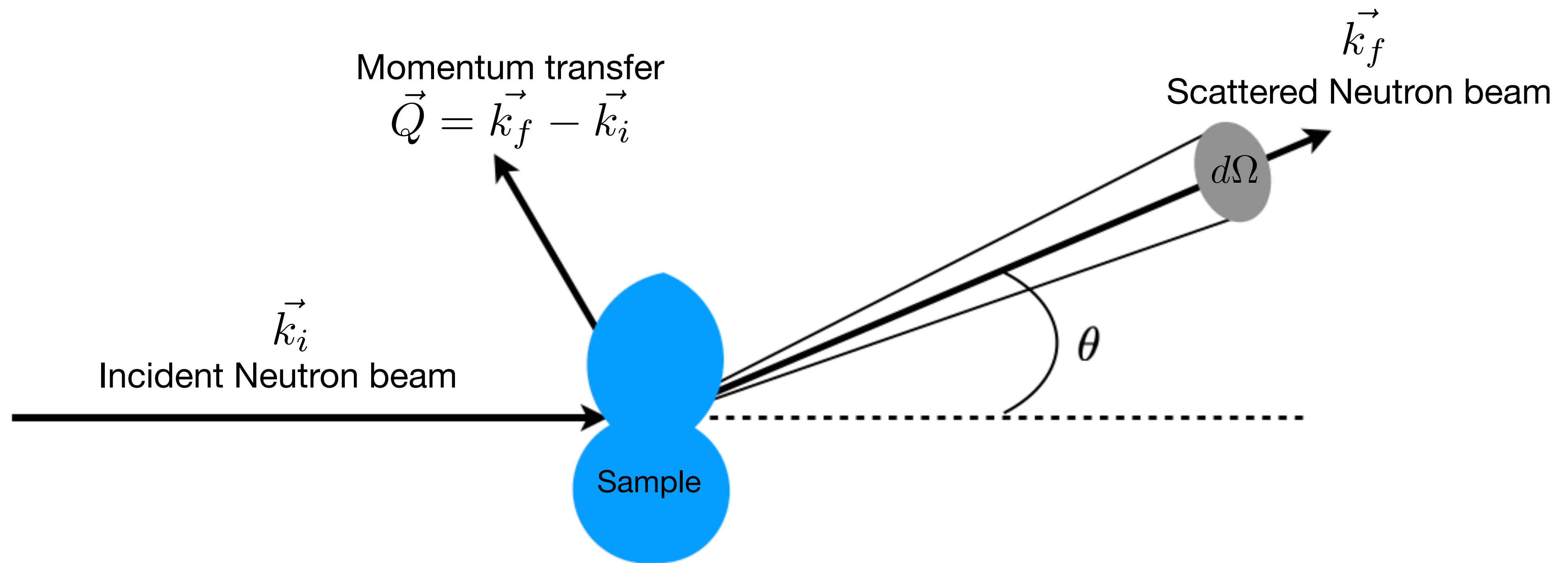
$\approx$



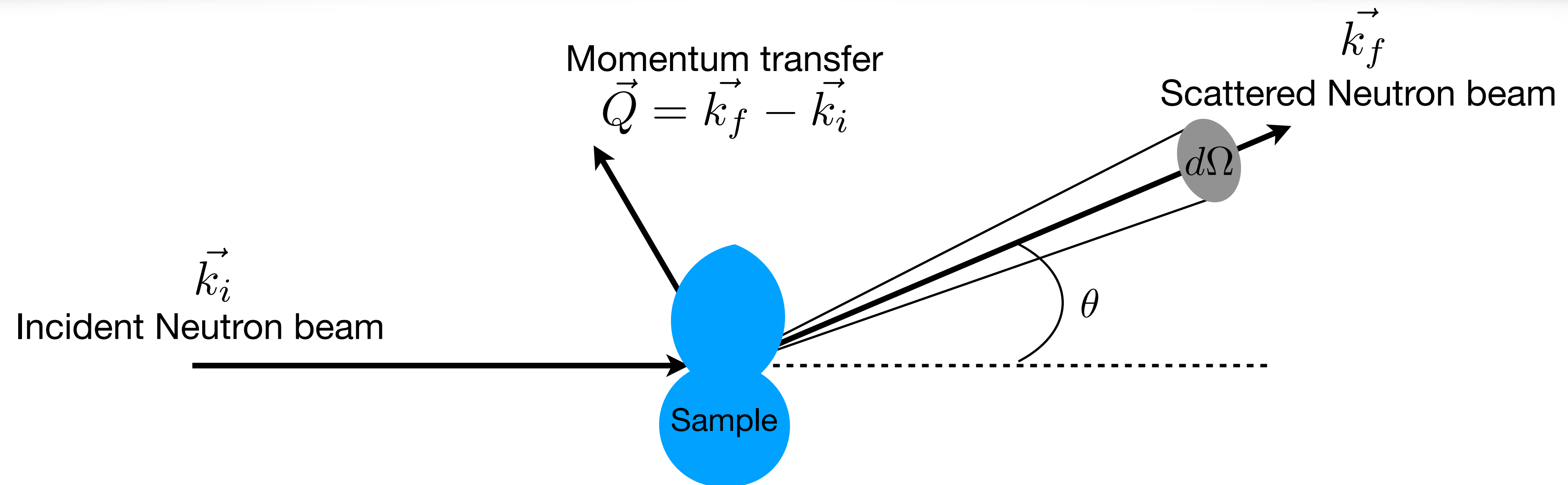
Generally, a neutron scattering experiment consists of counting the number of neutrons scattered by a sample in different directions.

For this talk, we assume elastic scattering only

$$\Delta E = 0$$



Neutron cross-sections are helpful to characterize the strength of interaction between the sample and the neutron



Total Cross-section

$$\sigma_{tot} = \frac{\text{Number of neutrons scattered in all directions per second}}{\text{Incident flux } (I_0)}$$

Units of area in barns (1 barn = 10^{-28} m²)

Differential Cross-section

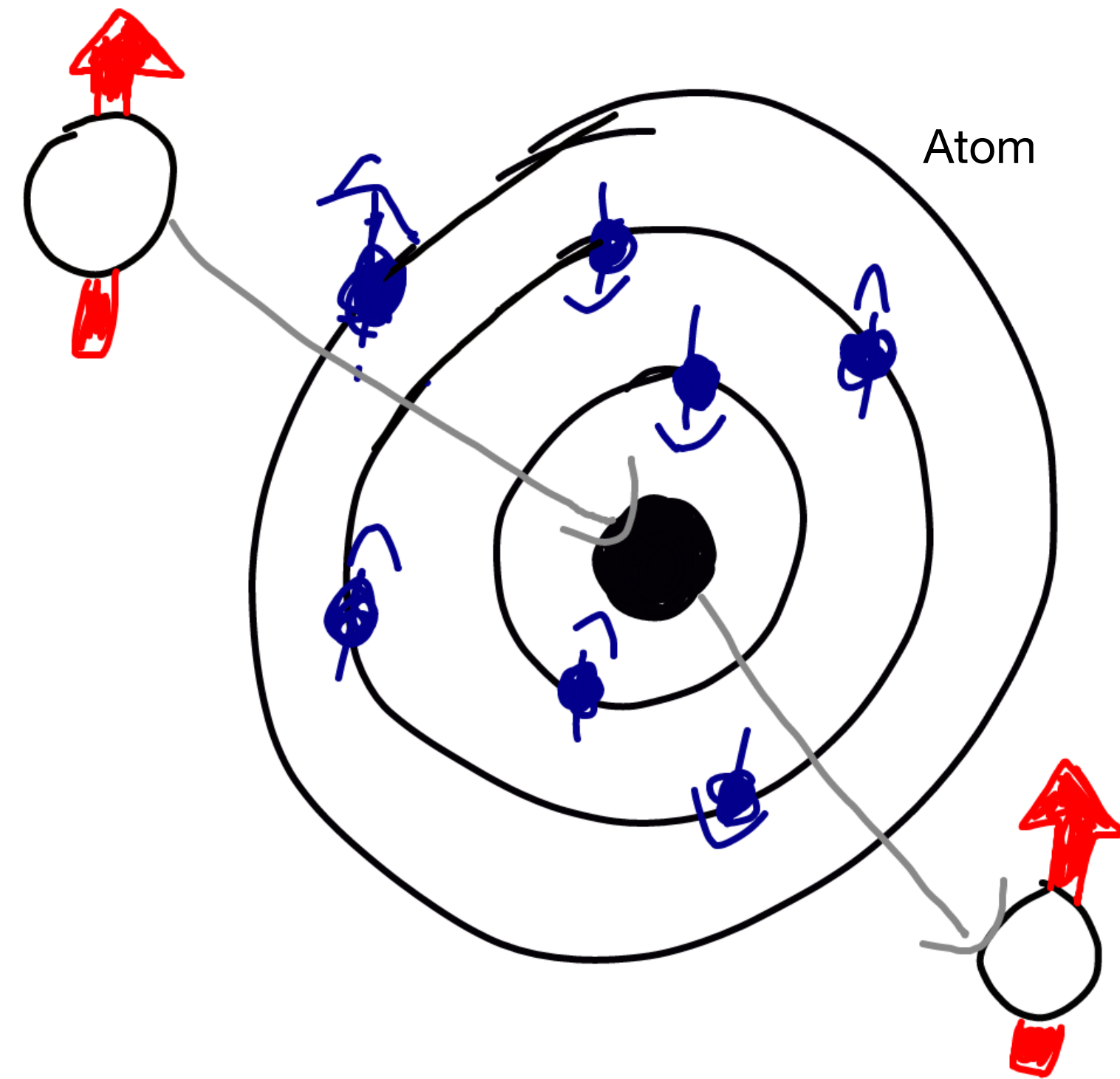
$$\frac{d\sigma}{d\Omega} = \frac{\text{Number of neutrons scattered per sec. into a solid angle } d\Omega}{\text{Incident flux } (I_0)}$$

Units are barns/steradian

** A macroscopic diff. cross-section is often used, which divides the diff. cross-section by the volume of the sample.

A neutron interacts with the atoms within a material via 2 different ways

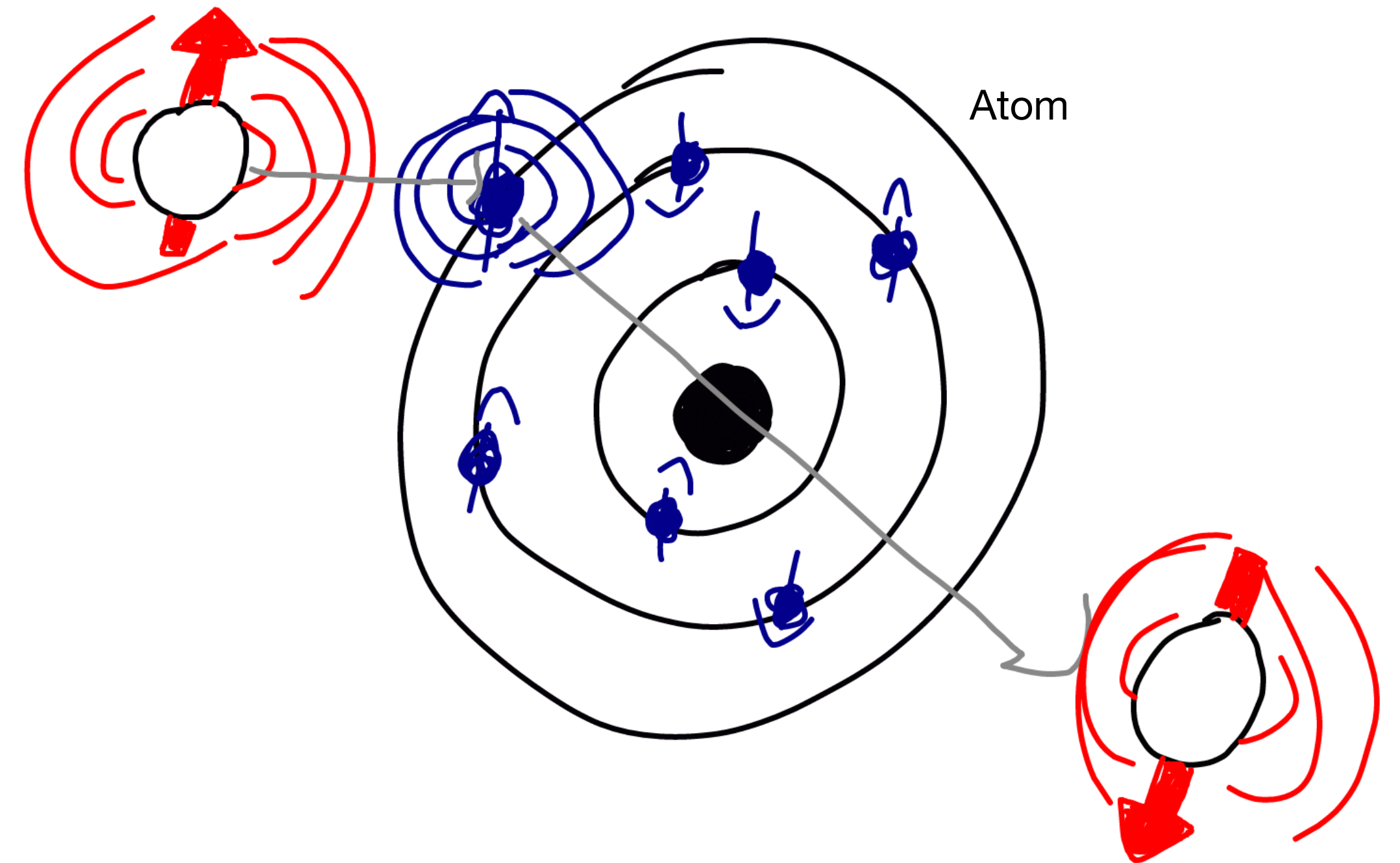
Nuclear Interaction



$$\hat{V}_N(\mathbf{r}_n, \mathbf{R}_N) = -\frac{2\pi\hbar^2}{m_n} b \delta(\mathbf{r}_n - \mathbf{R}_N)$$

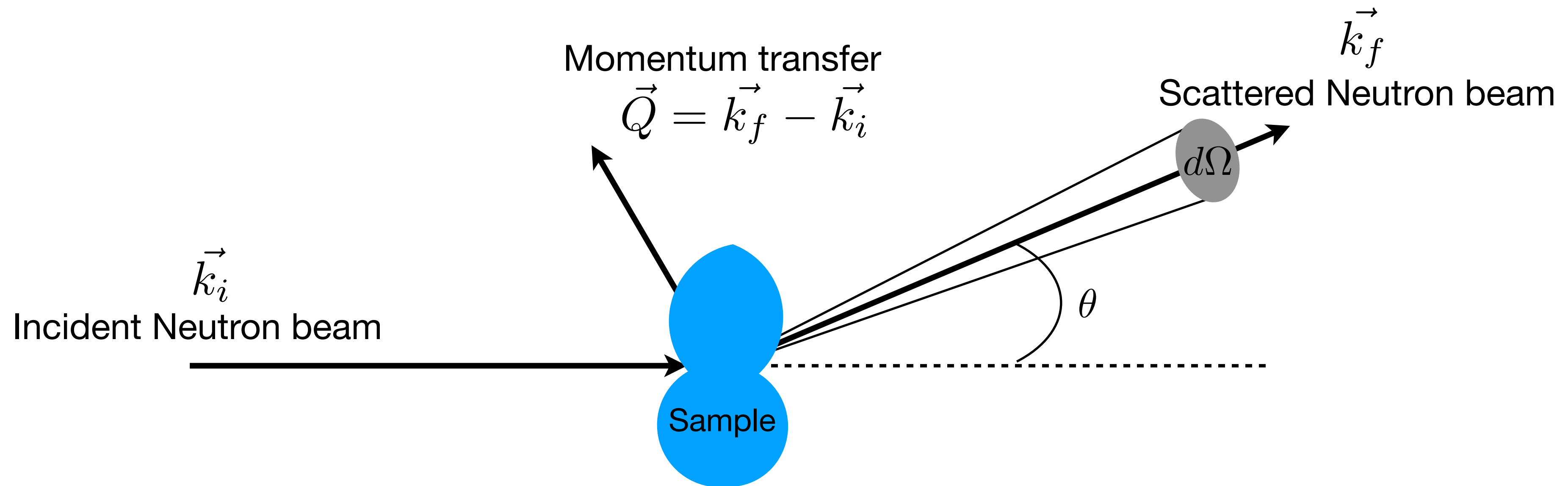
b: Neutron Scattering length ($\sim 1\text{fm} = 1\text{e-}15\text{ m}$)

Magnetic Dipole-Dipole Interaction



$$\hat{V}_M(\vec{r}_n, \vec{r}_e) = -\vec{S}_n \cdot \vec{B}(\vec{r}_n - \vec{r}_e)$$

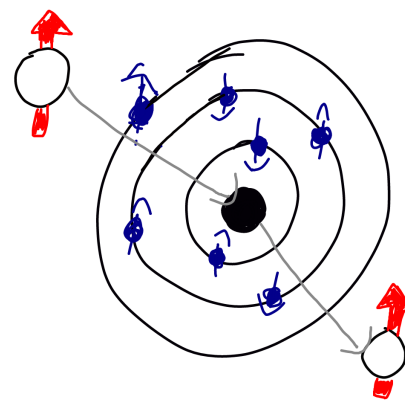
The neutron differential cross-section probes the Fourier transformation of the density (ρ) and magnetization (M) of a material



Nuclear Neutron diff. Cross-section

$$\frac{d\sigma}{d\Omega}(\vec{Q}) \propto \left| \int_V \rho(\vec{r}) e^{i\vec{Q} \cdot \vec{r}} d\vec{r} \right|^2$$

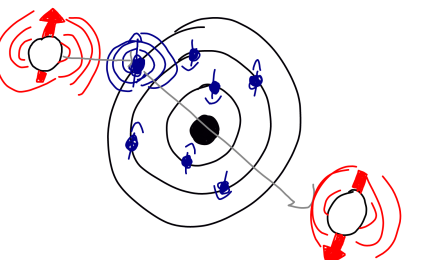
Nuclear Scattering Density



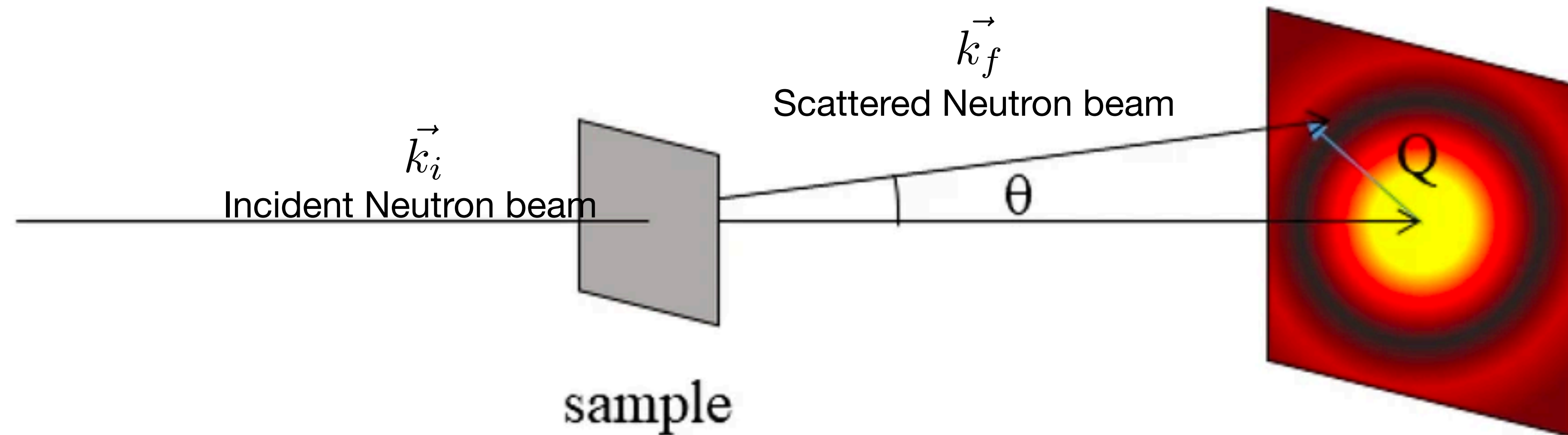
Magnetic Neutron diff. Cross-section

$$\frac{d\sigma}{d\Omega}(\vec{Q}) \propto \left| \int_V M_{\perp Q}(\vec{r}) e^{i\vec{Q} \cdot \vec{r}} d\vec{r} \right|^2$$

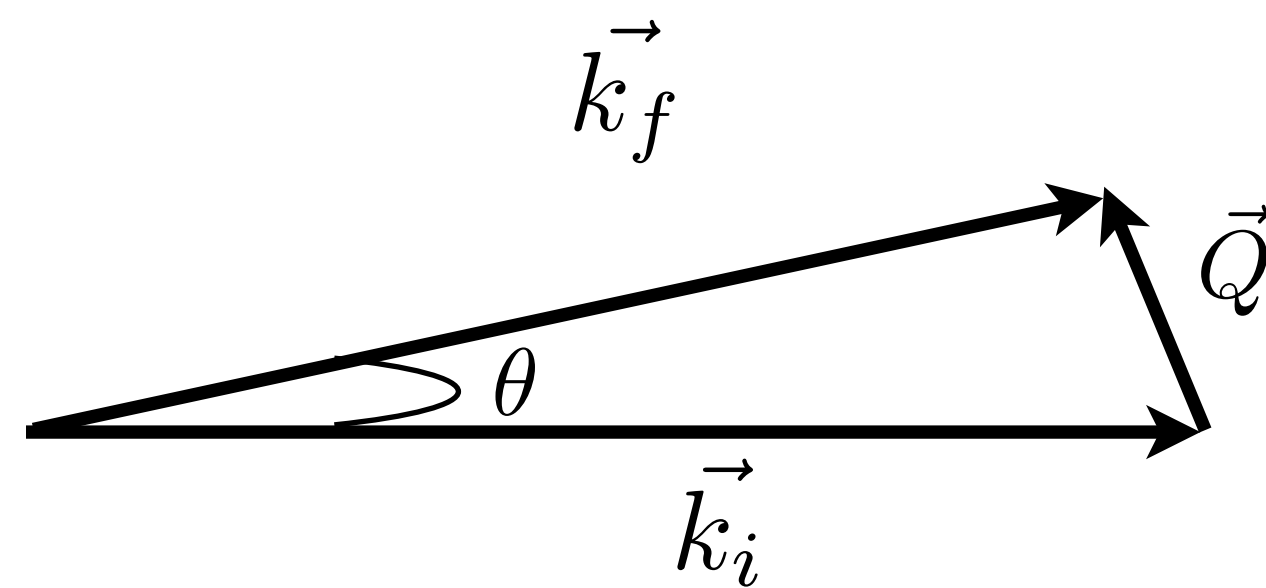
Magnetization Density



SANS: Small-Angle (low Q) neutron scattering is a technique designed to probe “long” length scale structures.



$$Q = 2k \sin\left(\frac{\theta}{2}\right) \sim k\theta$$

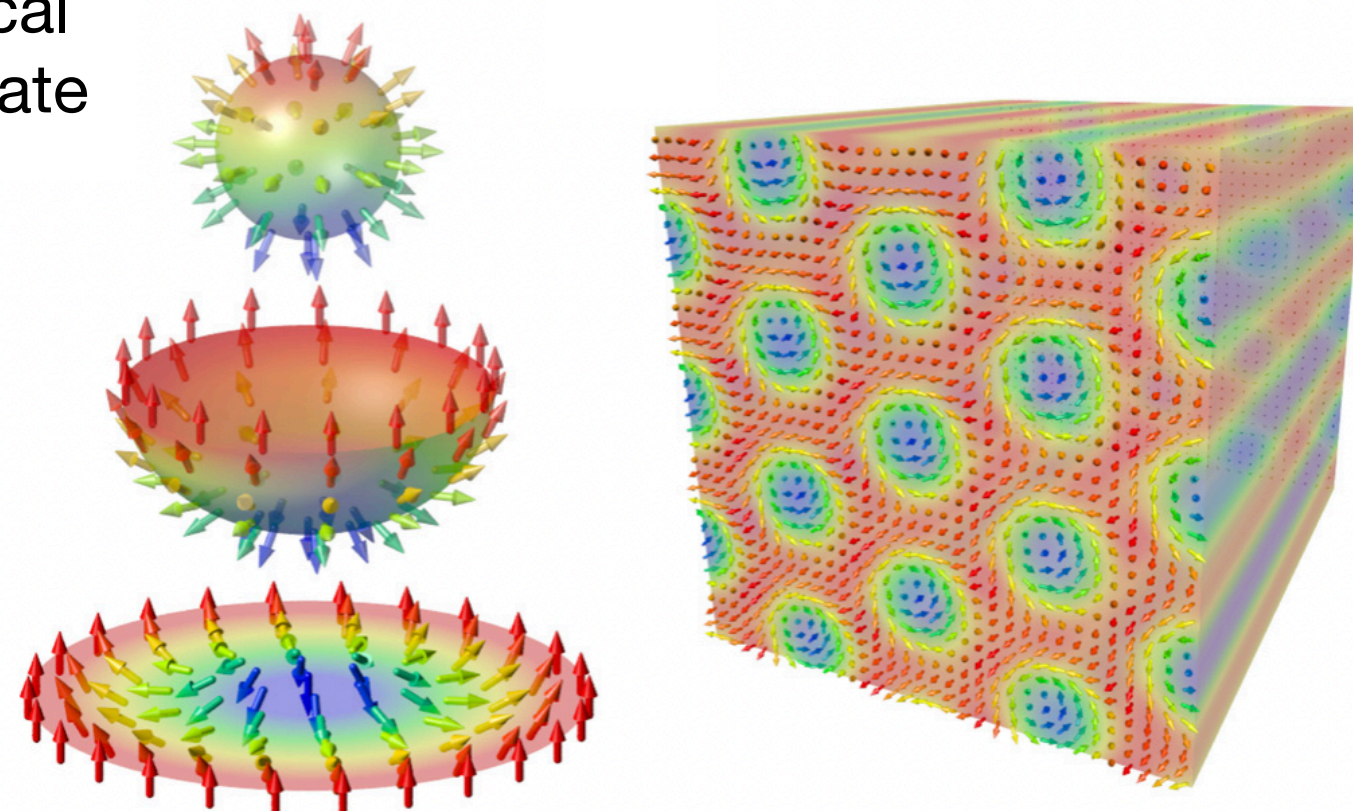


$$D = \frac{2\pi}{|\vec{Q}|}$$

D (length scale) ~ 1 to 10 000 nm

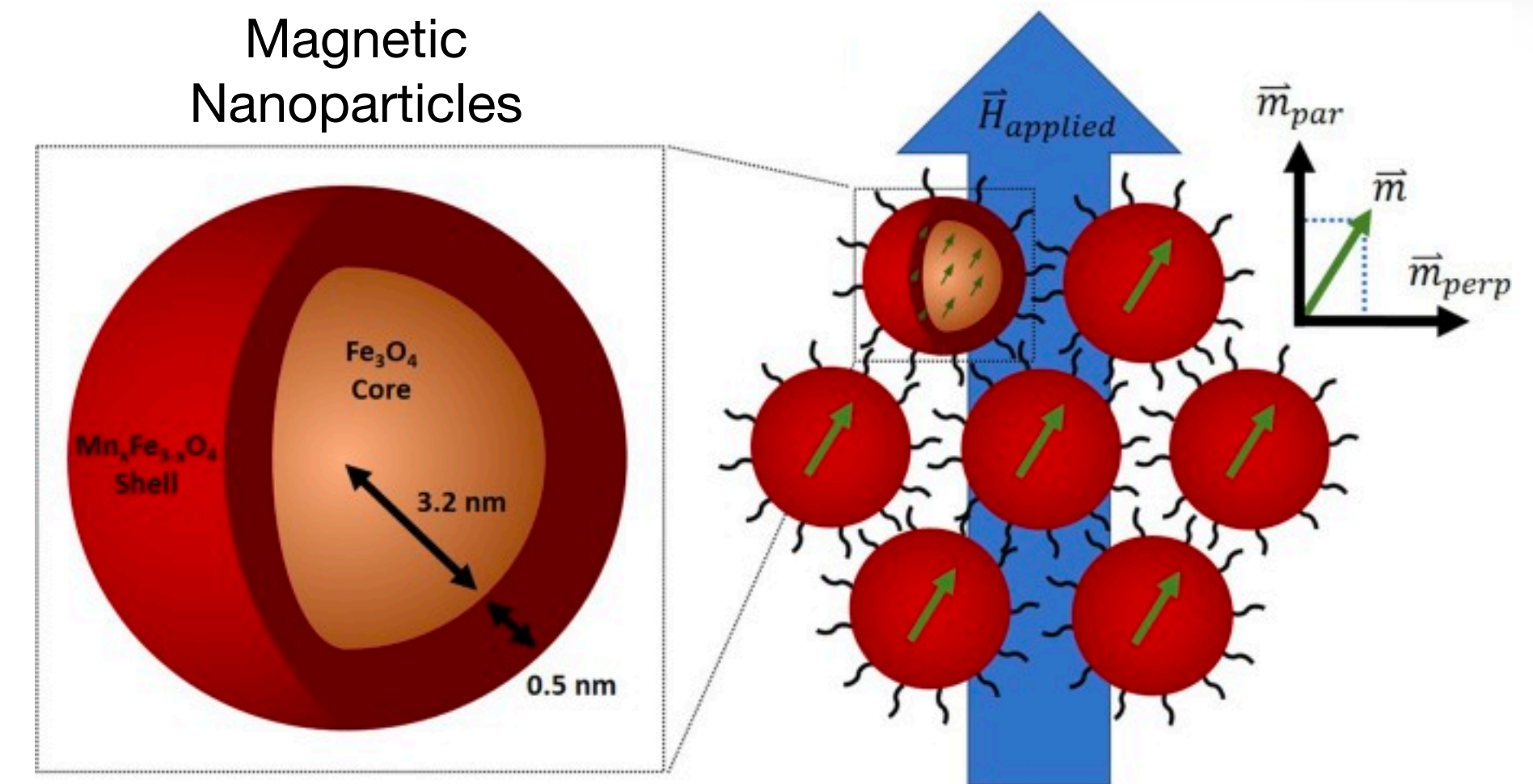
SANS is useful for various hard condensed matter systems

Topological
Multi-**q** state



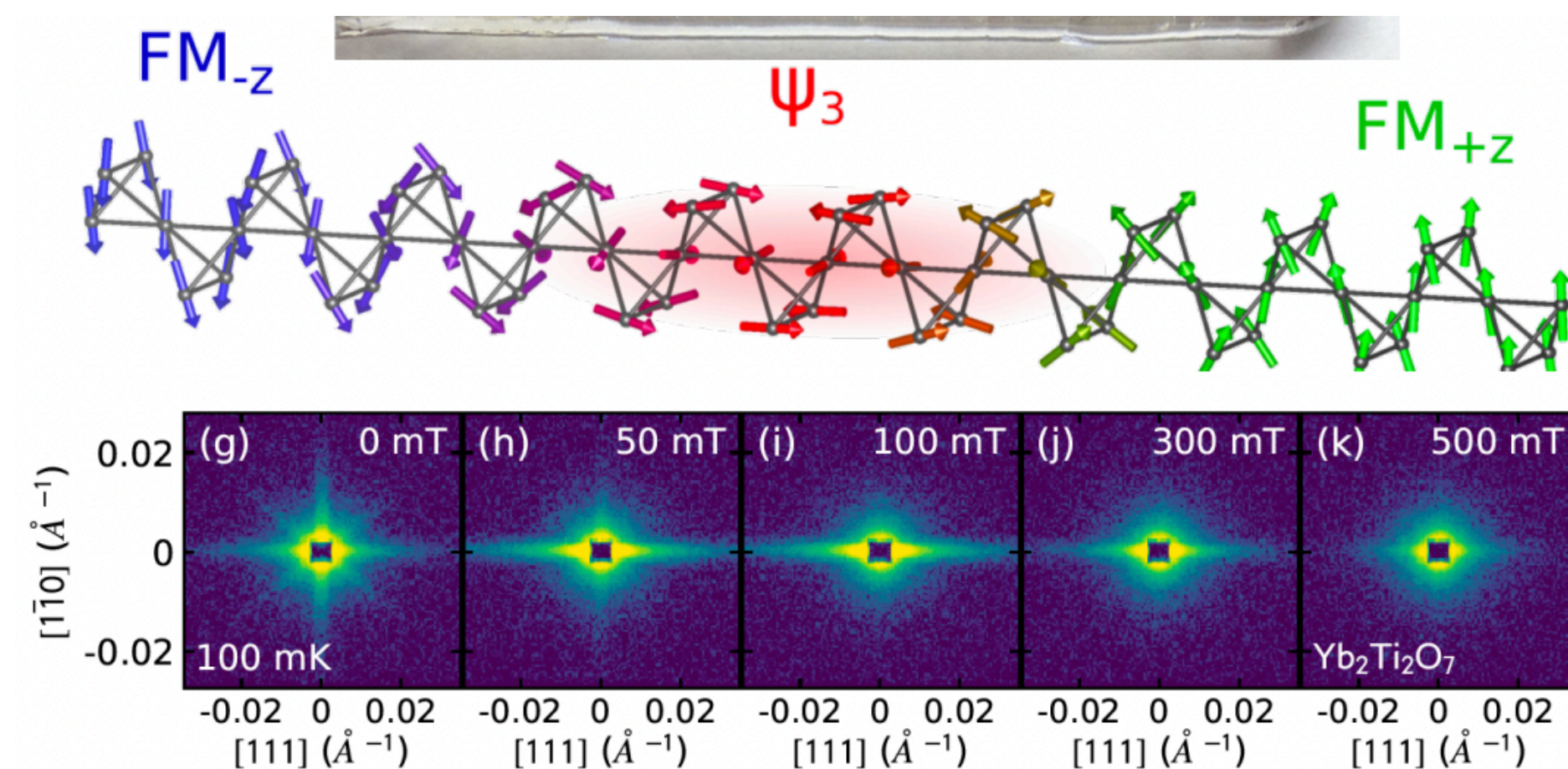
K. Everschor-Sitte et al., J. Appl. Phys. 115, 172602(2014)

Magnetic
Nanoparticles



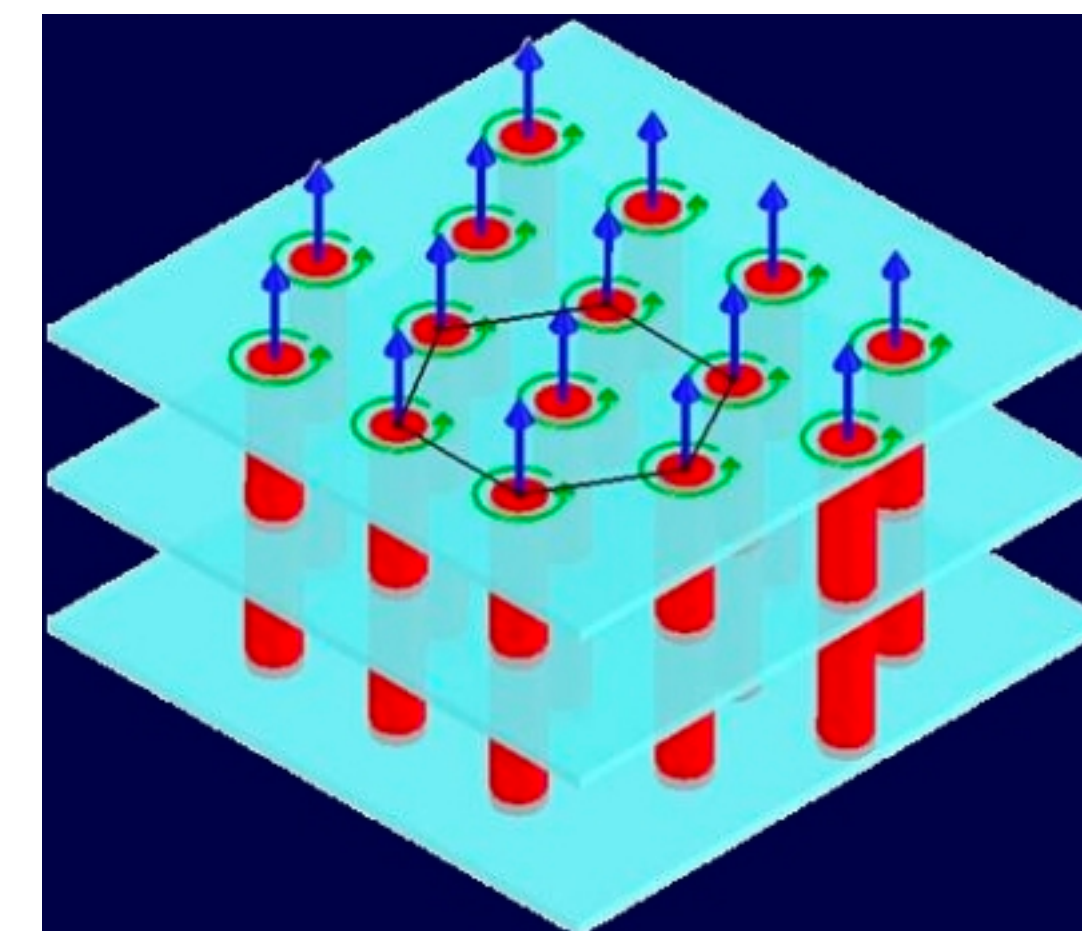
S. Oberdick et al., Sci. Reports 8, 3425 (2018)

Magnetic domain Walls



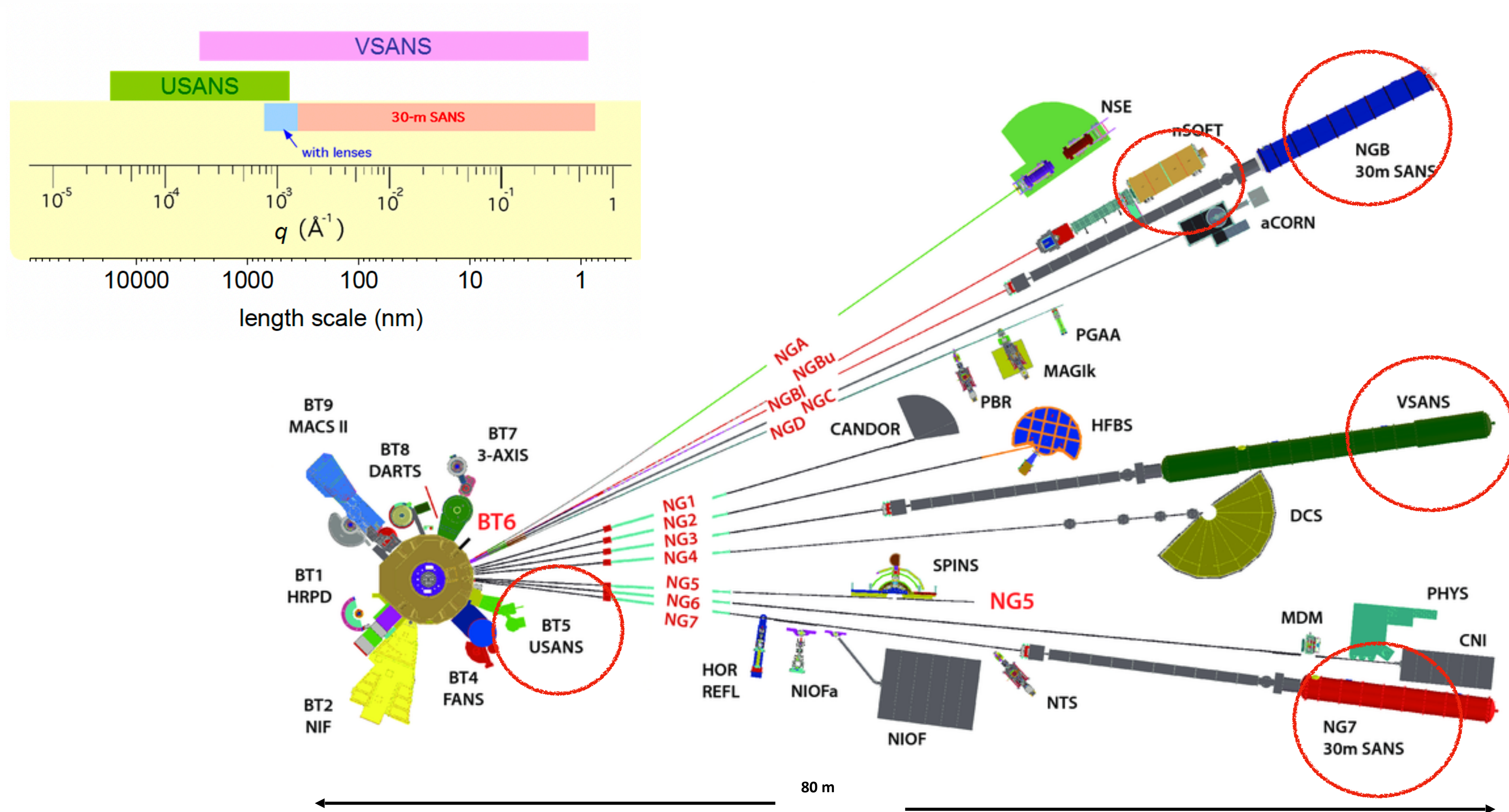
A. Scheie et al., PNAS 117 (44) 2020

Flux lattice
In superconductors

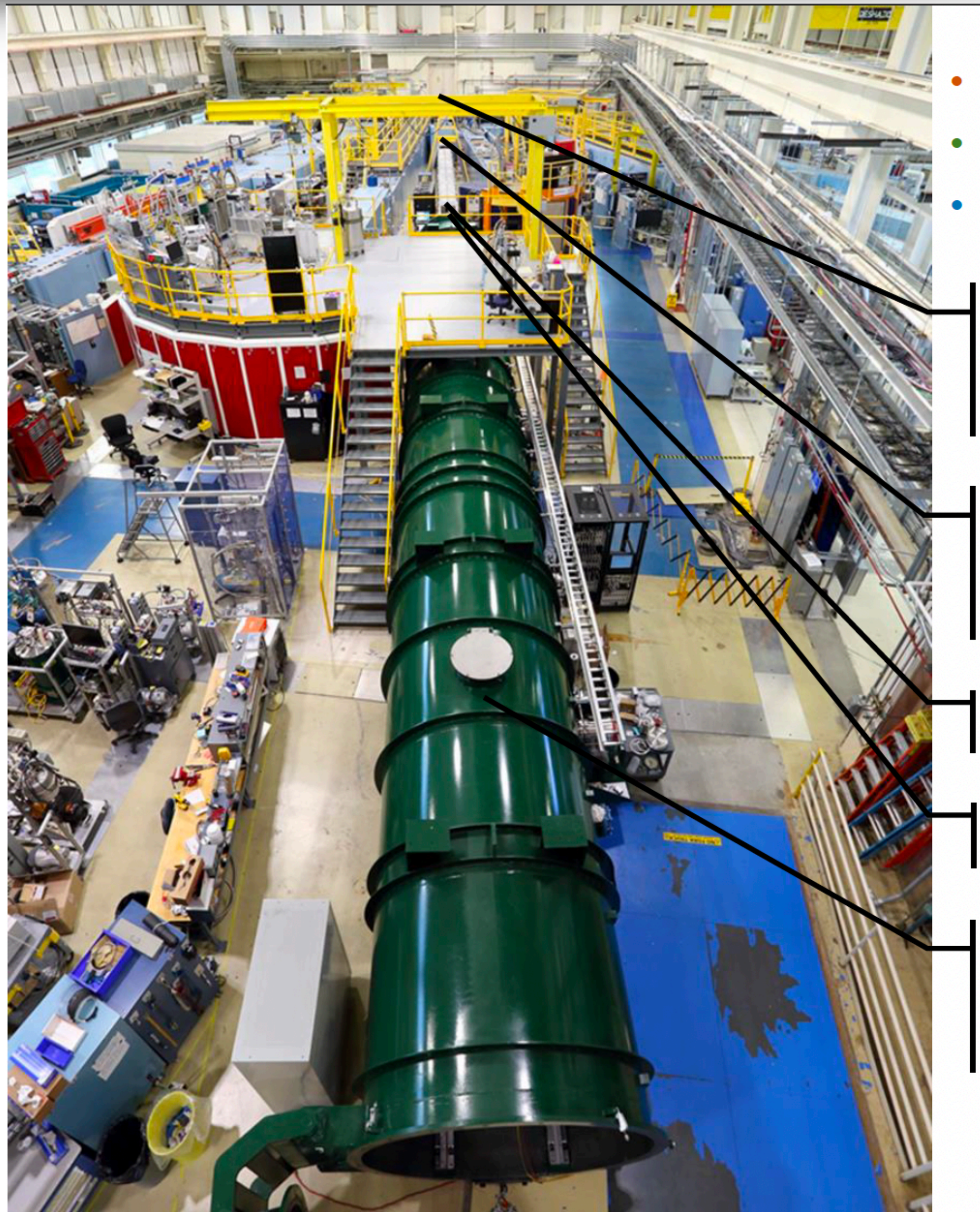


J. Hoffman STM Lab

NCNR is the host of 5 different SANS instruments



V-SANS is an extremely versatile neutron instrument



Various ways to select a specific distribution of incident neutrons wavelength

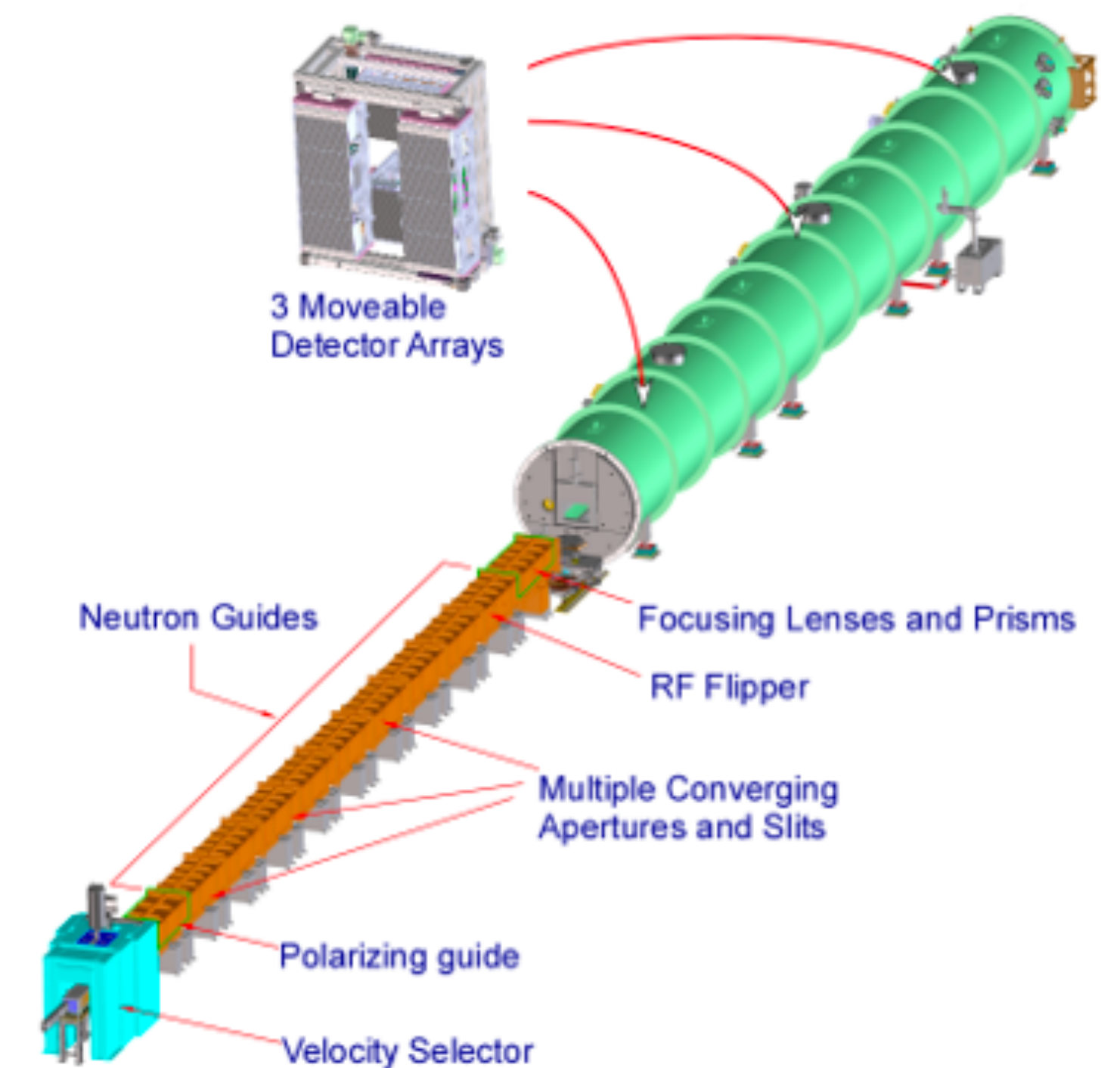
Multiple beam collimation options

Polarization capability

Large sample area

Three movable detectors

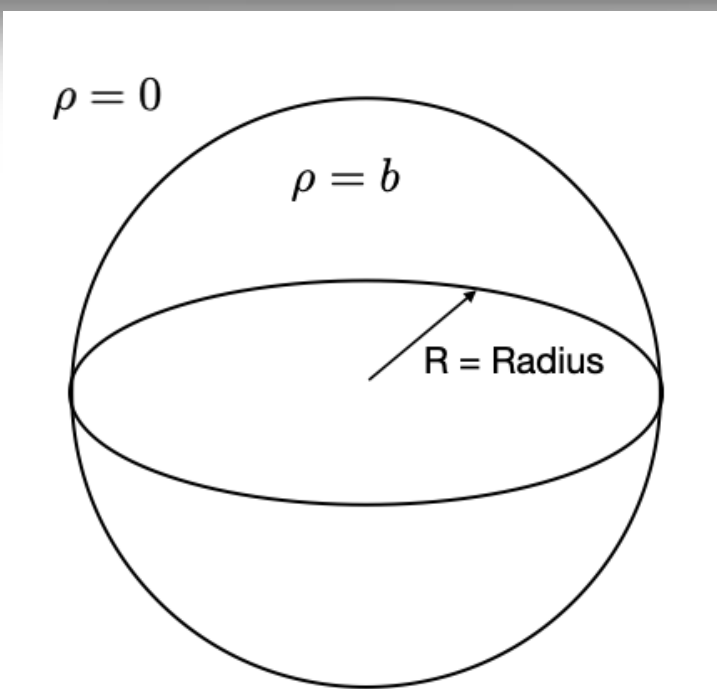
Barker J, et al. *J Appl. Cryst.* **55**(2) 271 (2022)



$Q \sim 2\text{e-}4 \text{ to } 0.7 \text{ \AA}^{-1}$

$D \sim 0.8 \text{ nm to } 3 \text{ microns}$

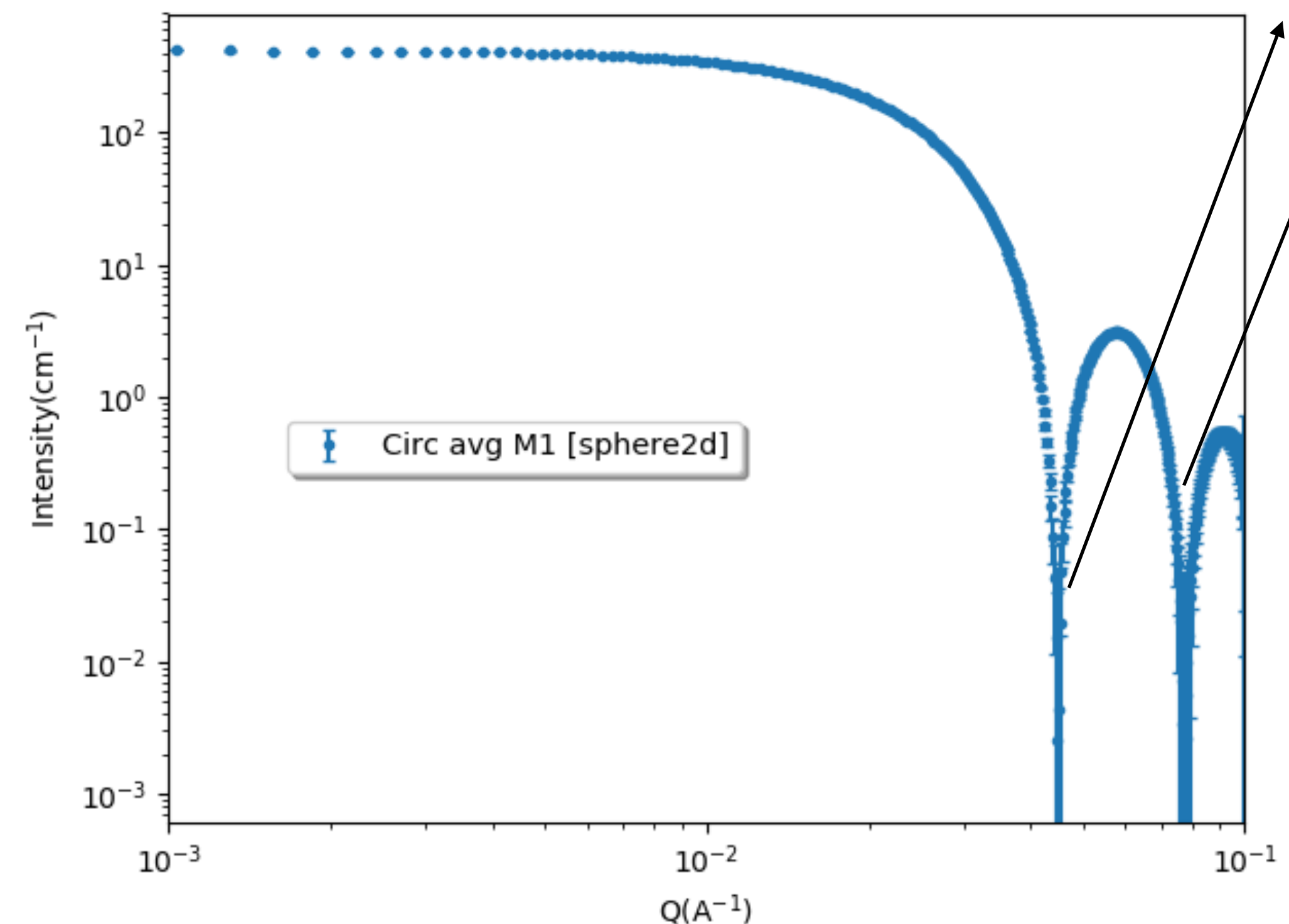
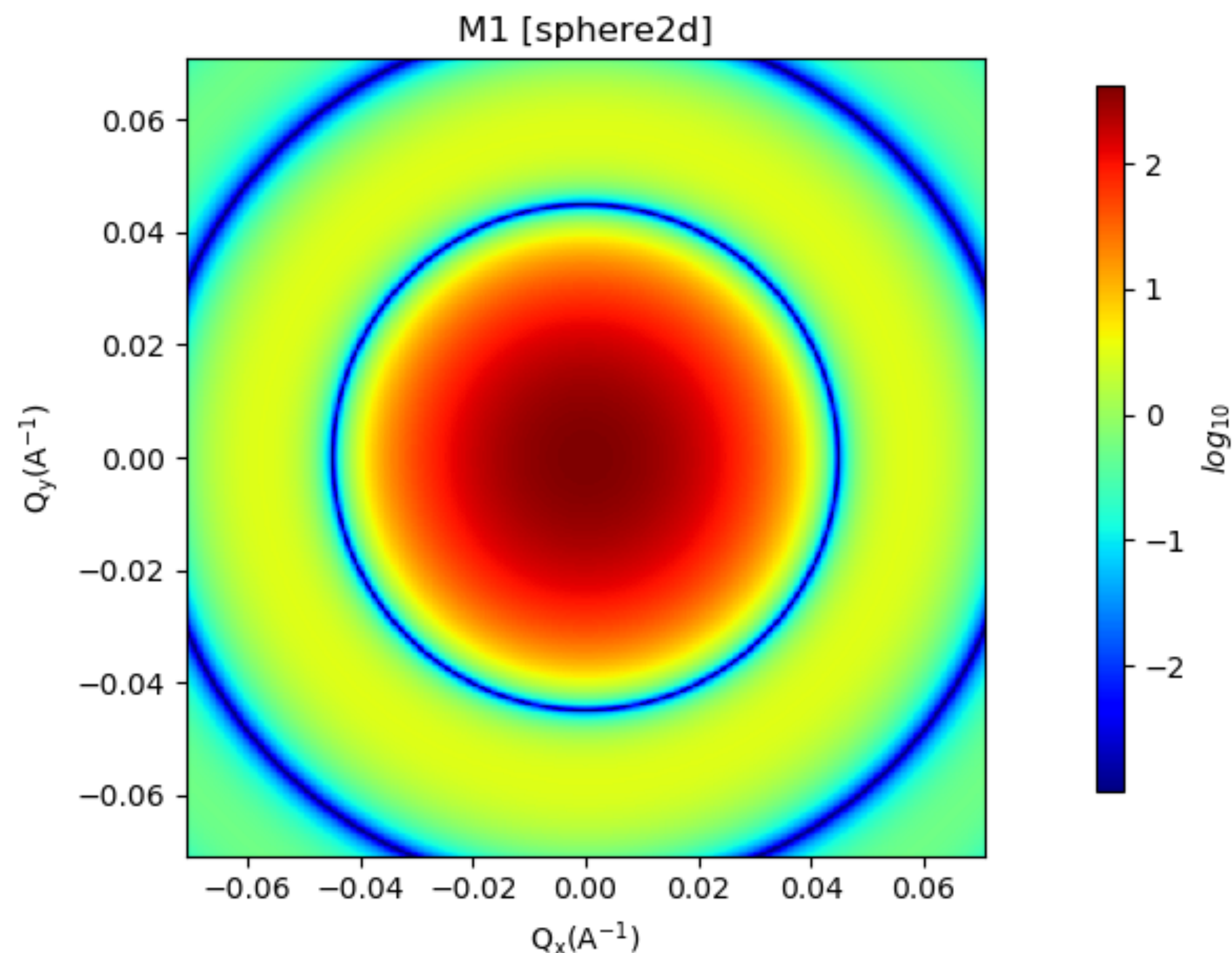
SANS from Nuclear Scattering: Scattering from uncorrelated (diluted) spheres



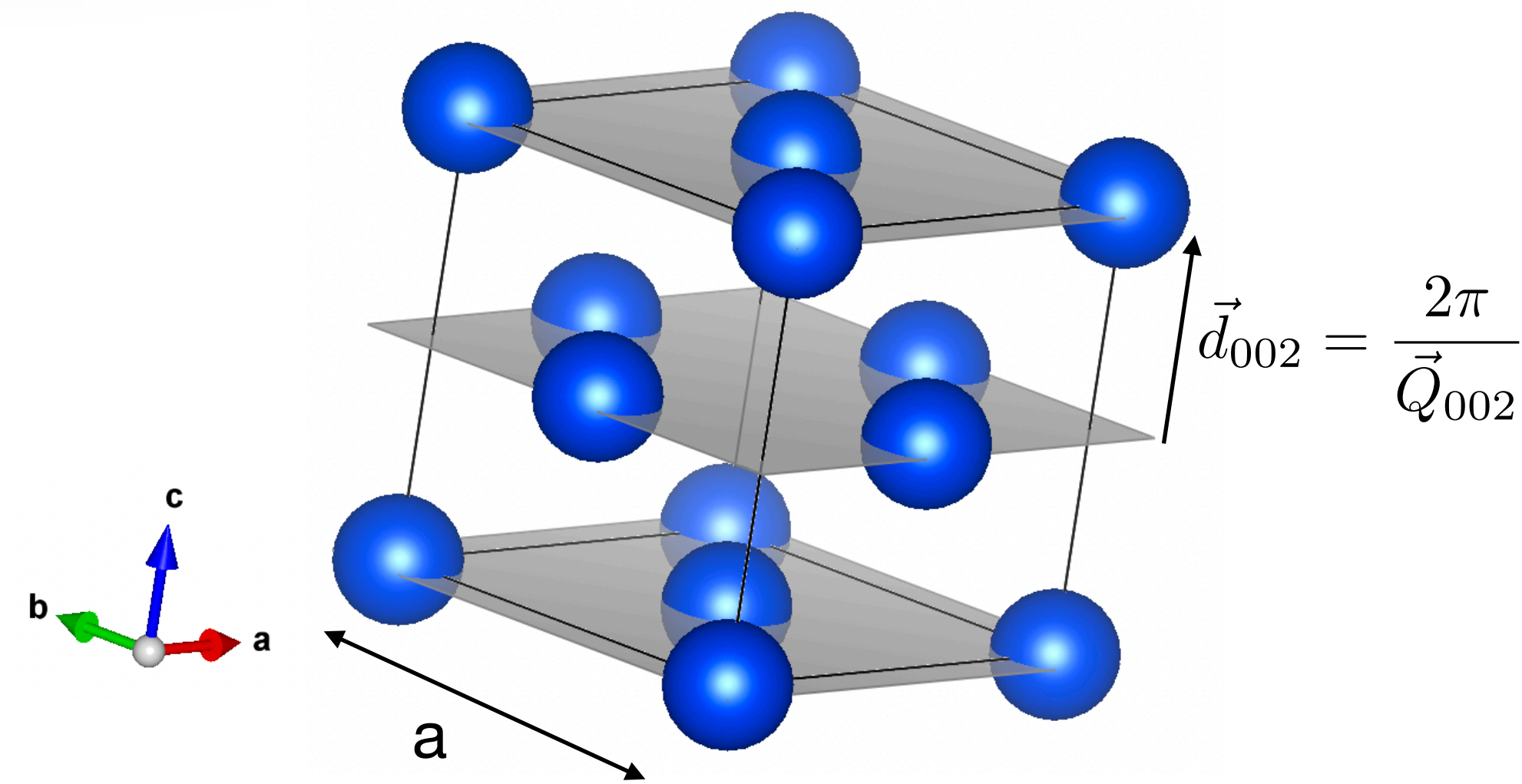
$$\frac{d\sigma}{d\Omega}(\vec{Q}) \propto \left| \int_V \rho(\vec{r}) e^{i\vec{Q} \cdot \vec{r}} d\vec{r} \right|^2 = 9b \left(\frac{\sin(QR) - QR \cos(QR)}{(QR)^3} \right)^2$$

Zeros of the function are located at:

$$QR \sim \pi(n + 1/2)$$



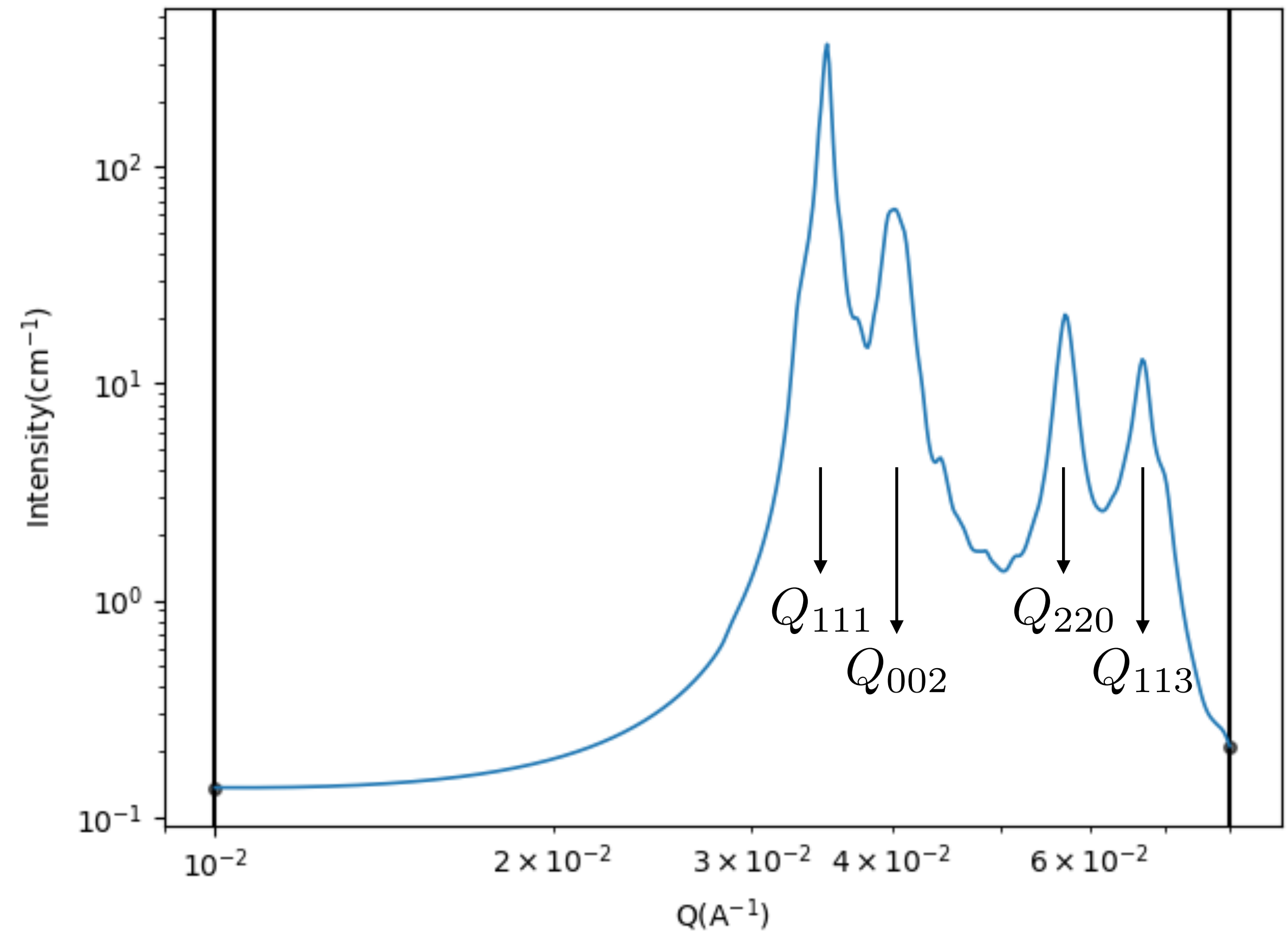
SANS from Nuclear Scattering: Scattering from a face-centered cubic (fcc) crystal lattice



$$\frac{d\sigma}{d\Omega}(\vec{Q}) \propto \left| \int_V \rho(\vec{r}) e^{i\vec{Q} \cdot \vec{r}} d\vec{r} \right|^2$$

Produce scattering peaks (Bragg reflections) indexed by

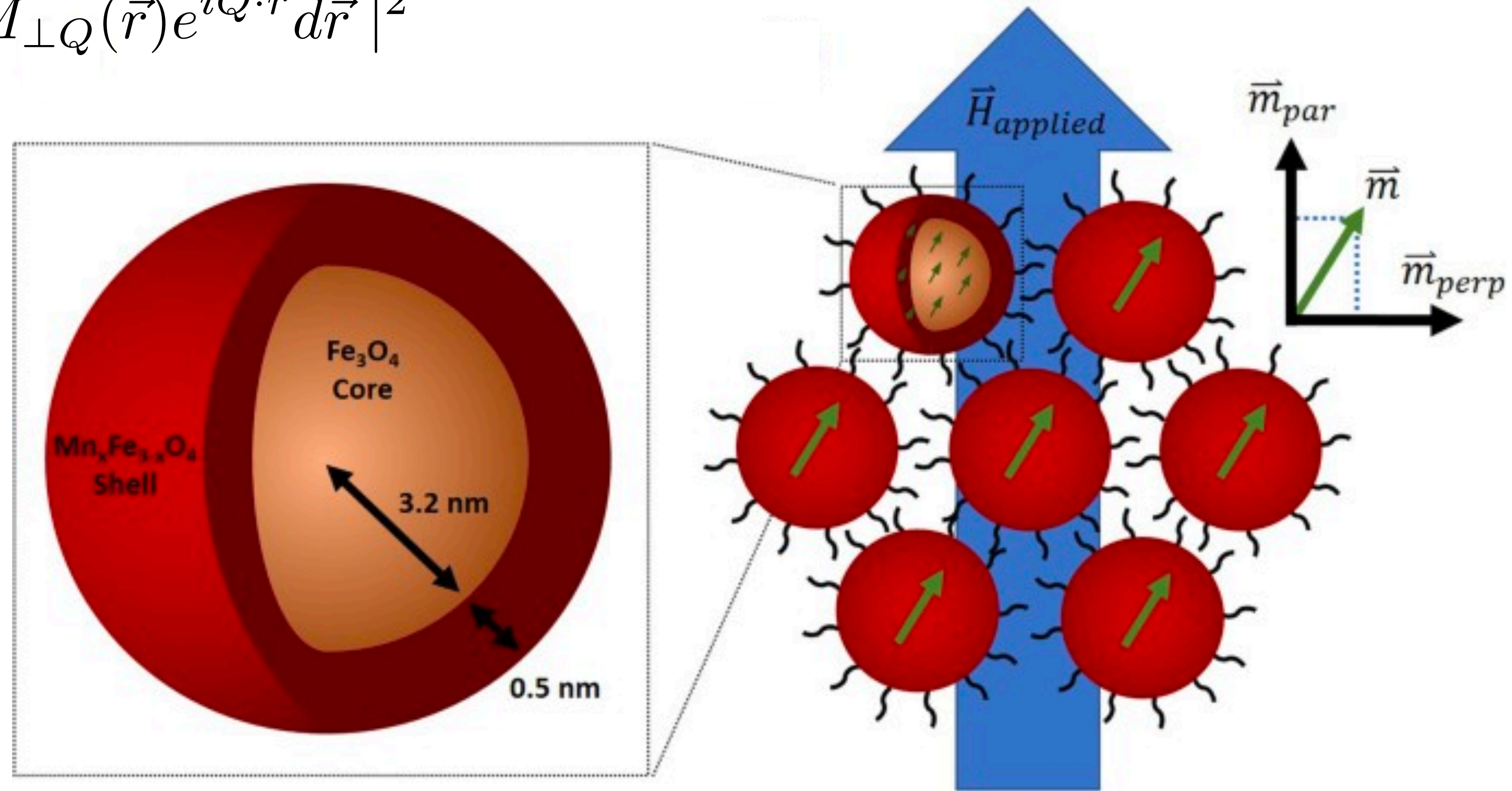
$$|Q_{Bragg}| = \frac{2\pi}{a} \sqrt{H^2 + K^2 + L^2}$$



For the fcc lattice, the peaks have finite intensity if the integers H,K, and L are all odd or all even.

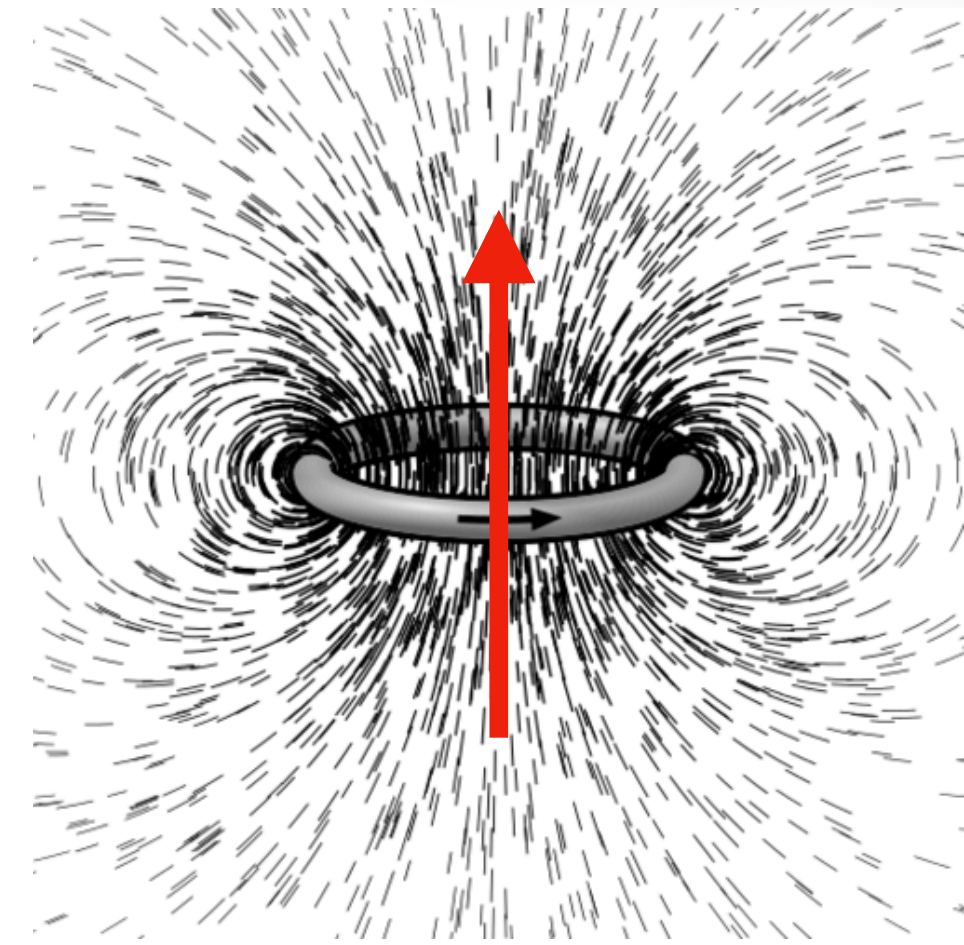
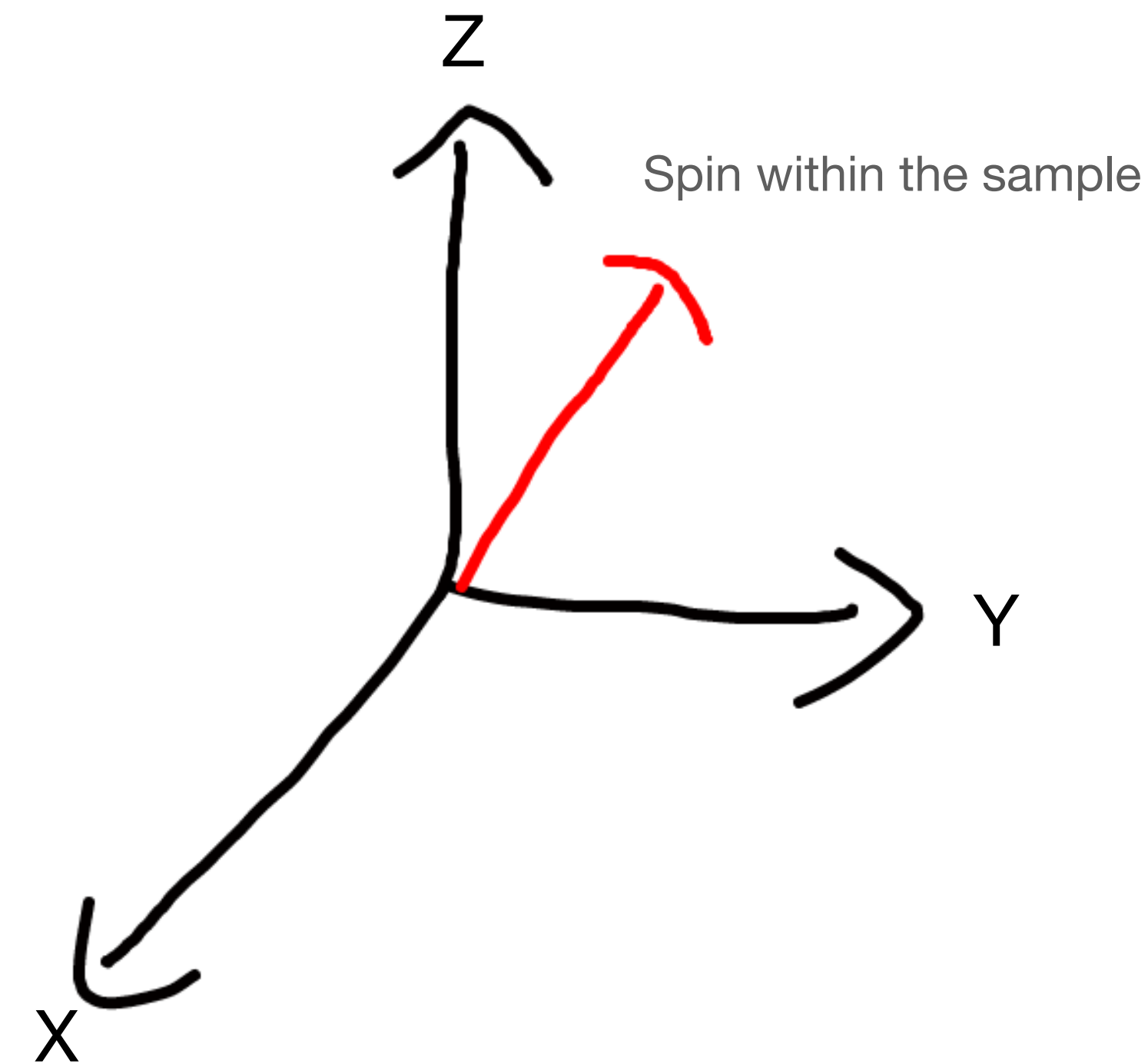
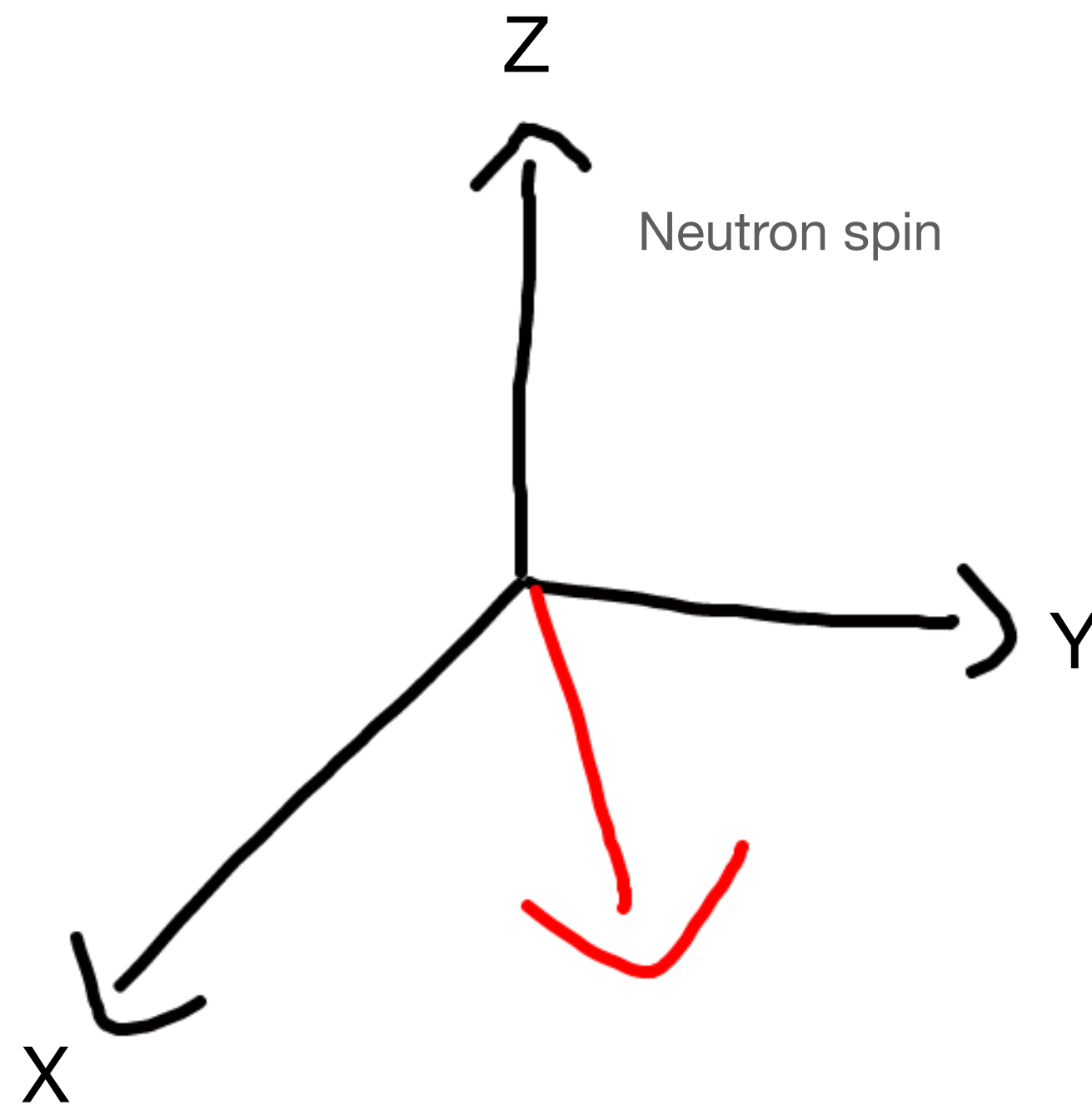
SANS from Magnetic Scattering

$$\frac{d\sigma}{d\Omega}(\vec{Q}) \propto \left| \int_V M_{\perp Q}(\vec{r}) e^{i\vec{Q} \cdot \vec{r}} d\vec{r} \right|^2$$

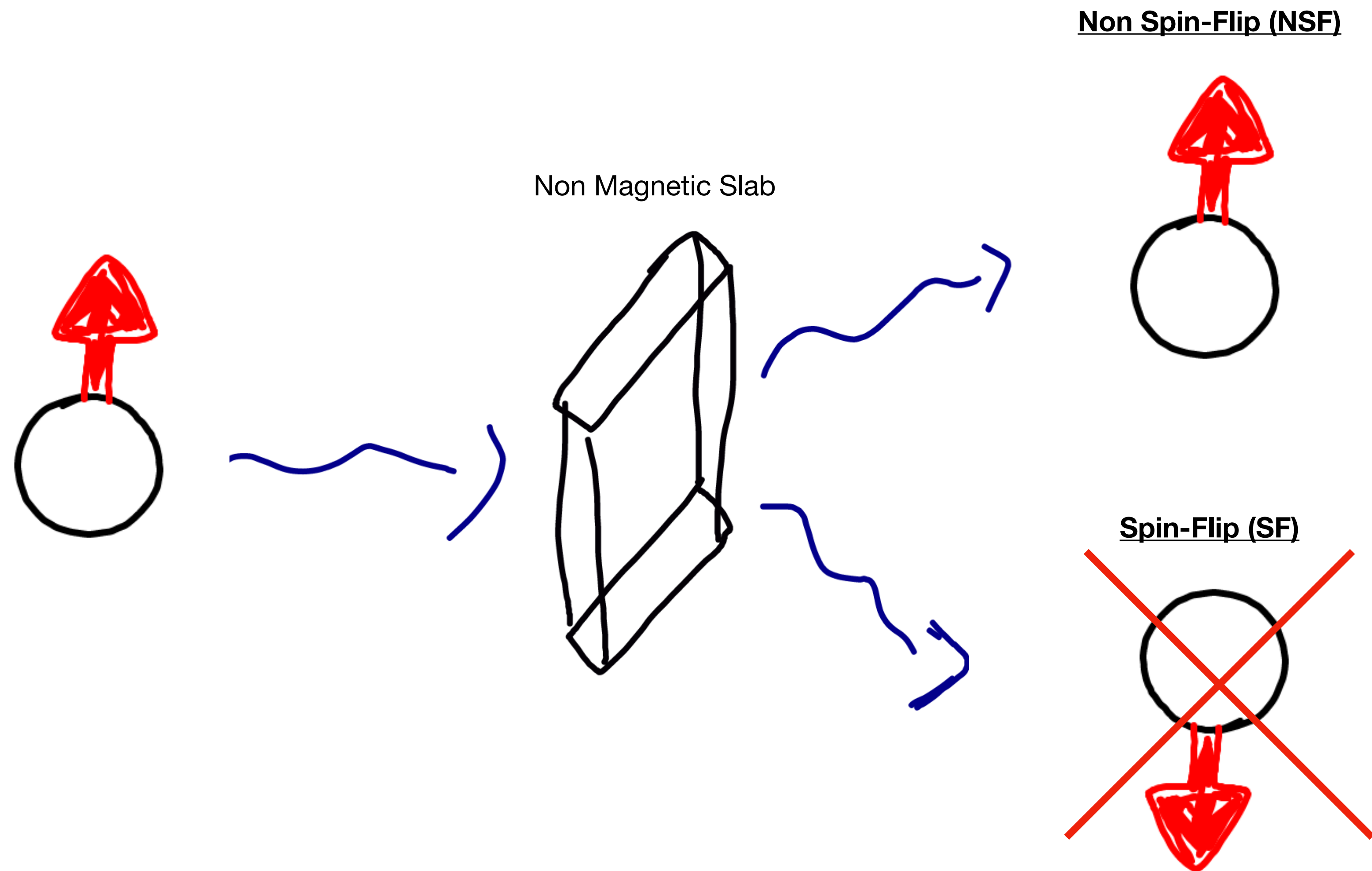


S. Oberdick et al., Sci. Reports 8, 3425 (2018)

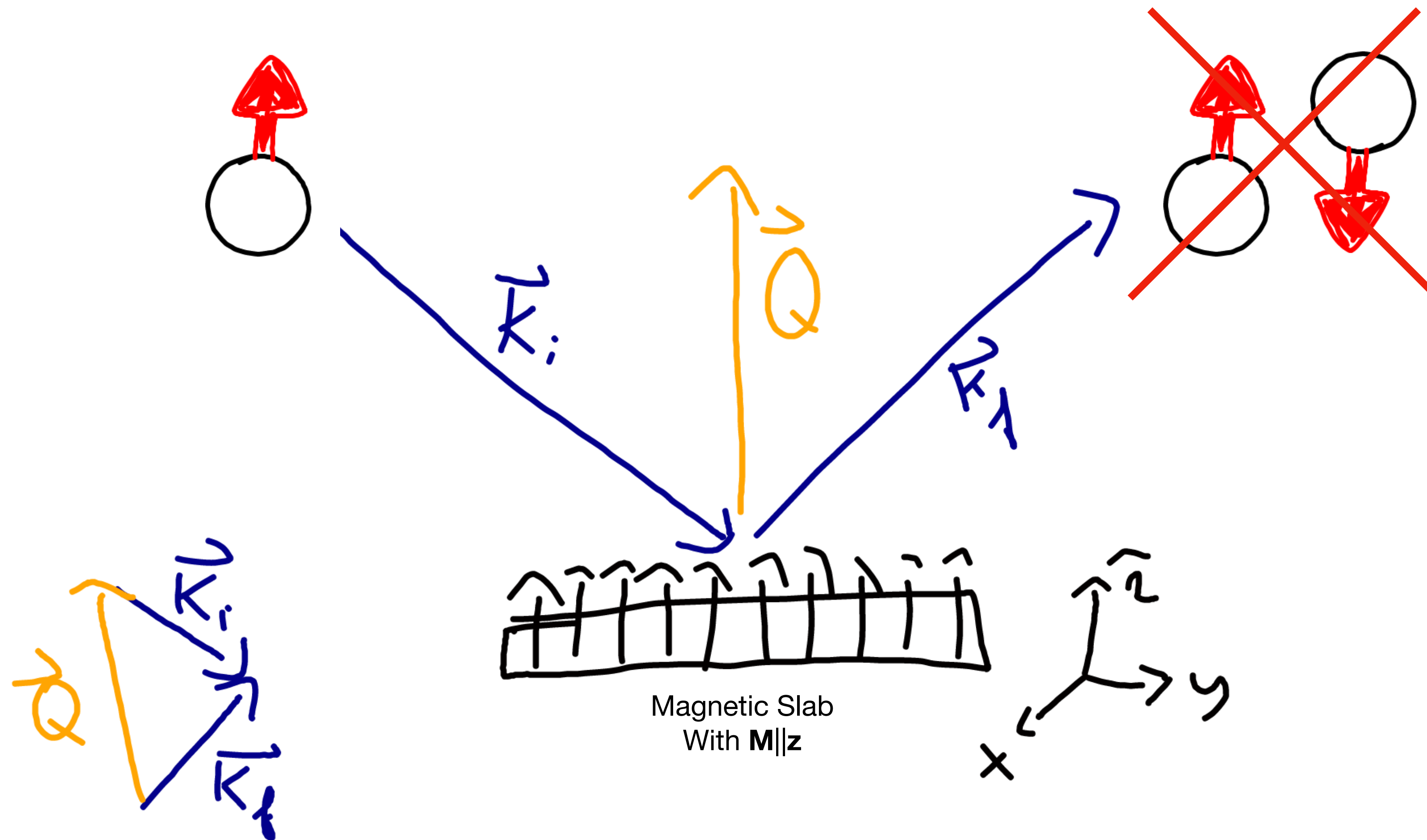
The magnetic interaction depends on the relative orientation of the spins within the probed sample



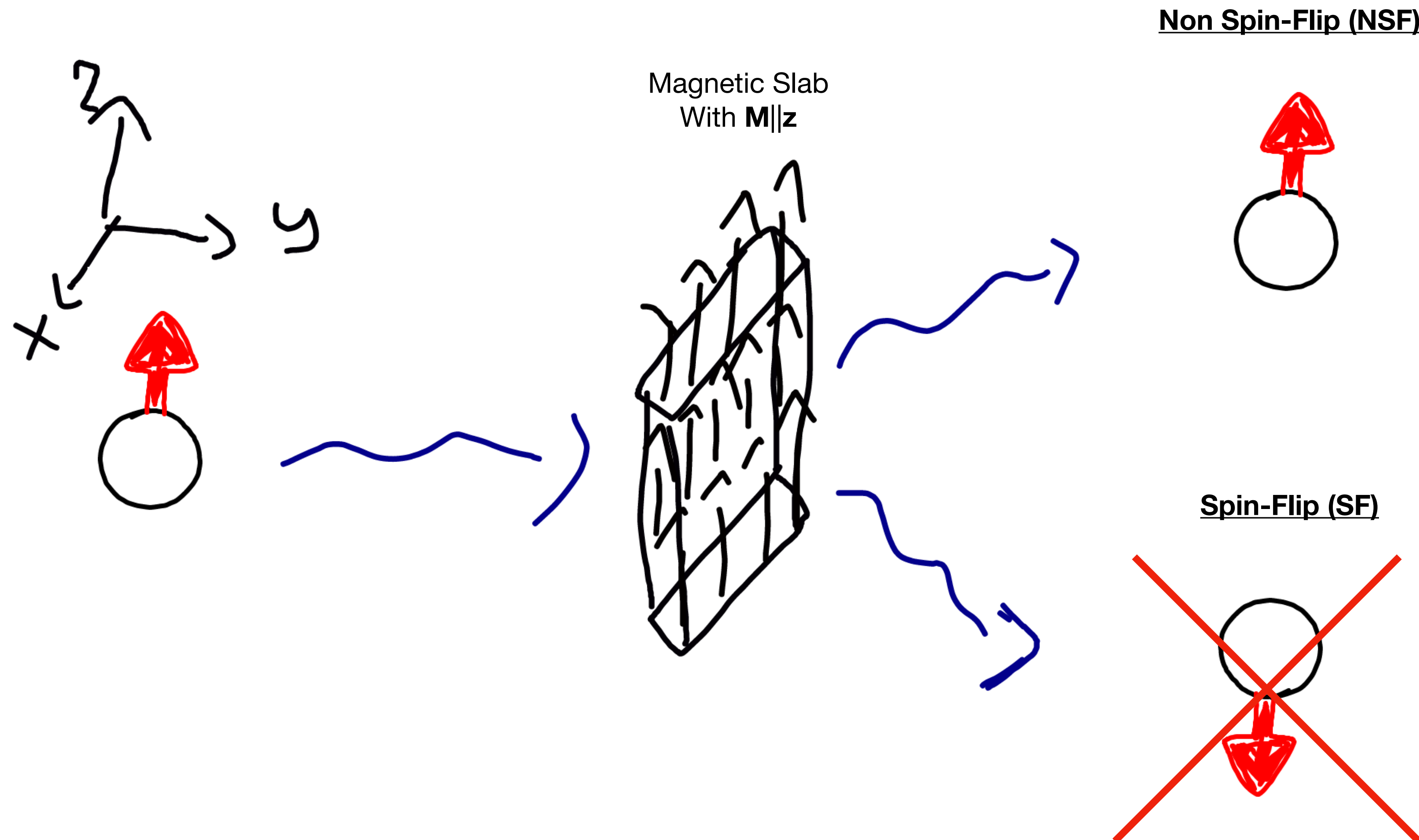
Rule 1: Nuclear scattering does not flip the neutron spin



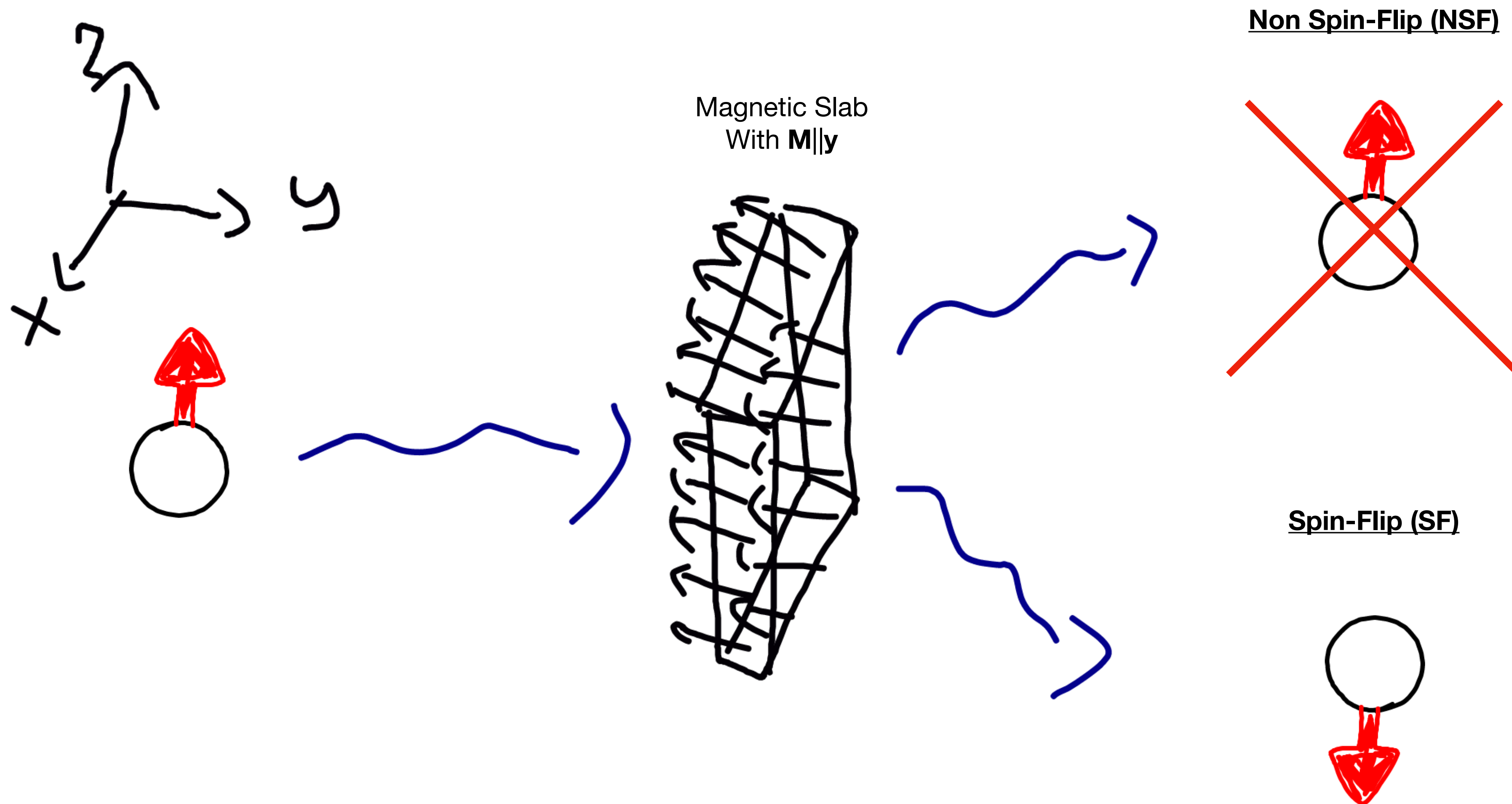
Rule 2: Magnetic scattering is only sensitive to magnetization perpendicular to the momentum transfer Q



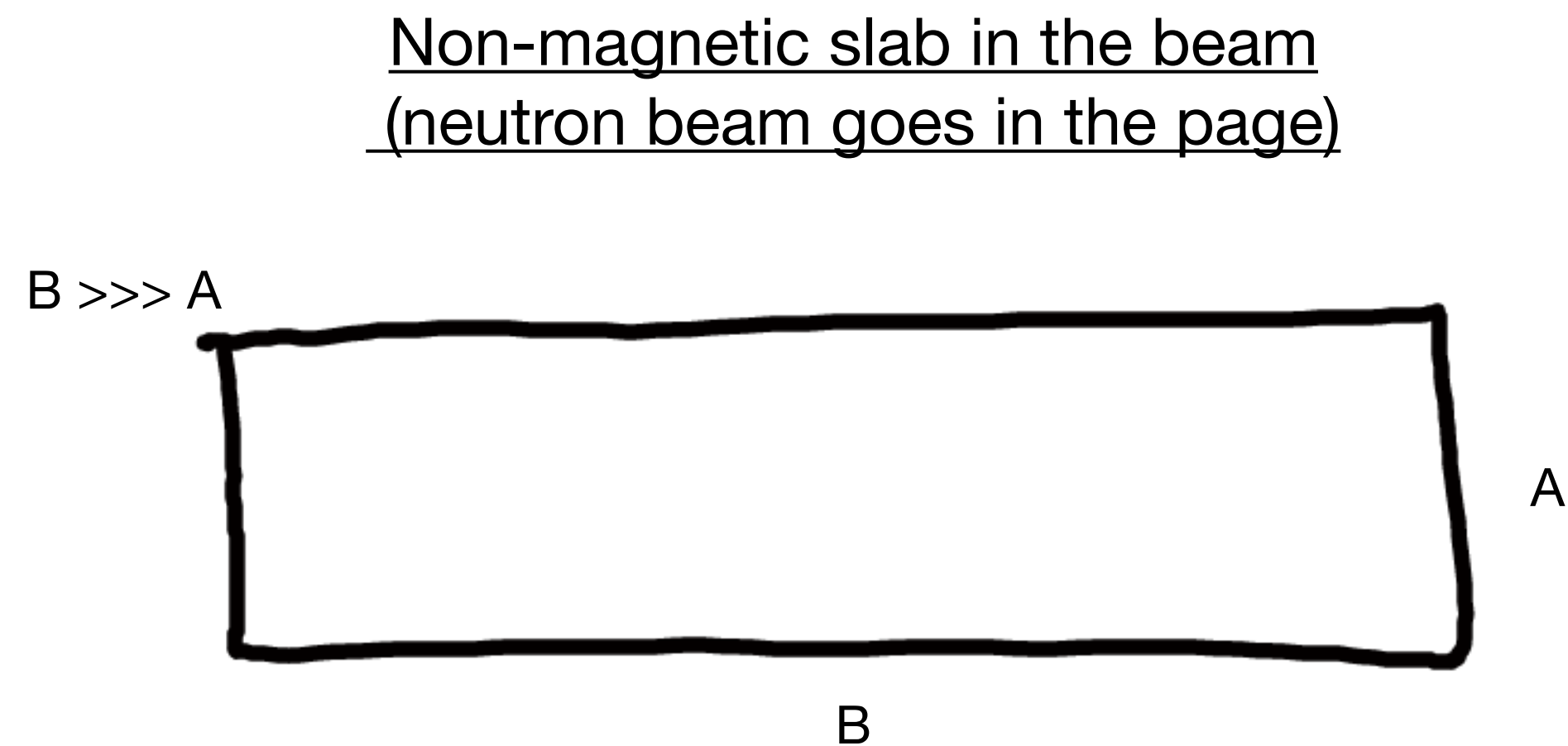
Rule 3: The non spin-flip scattering is only sensitive to magnetization parallel to the neutron spin



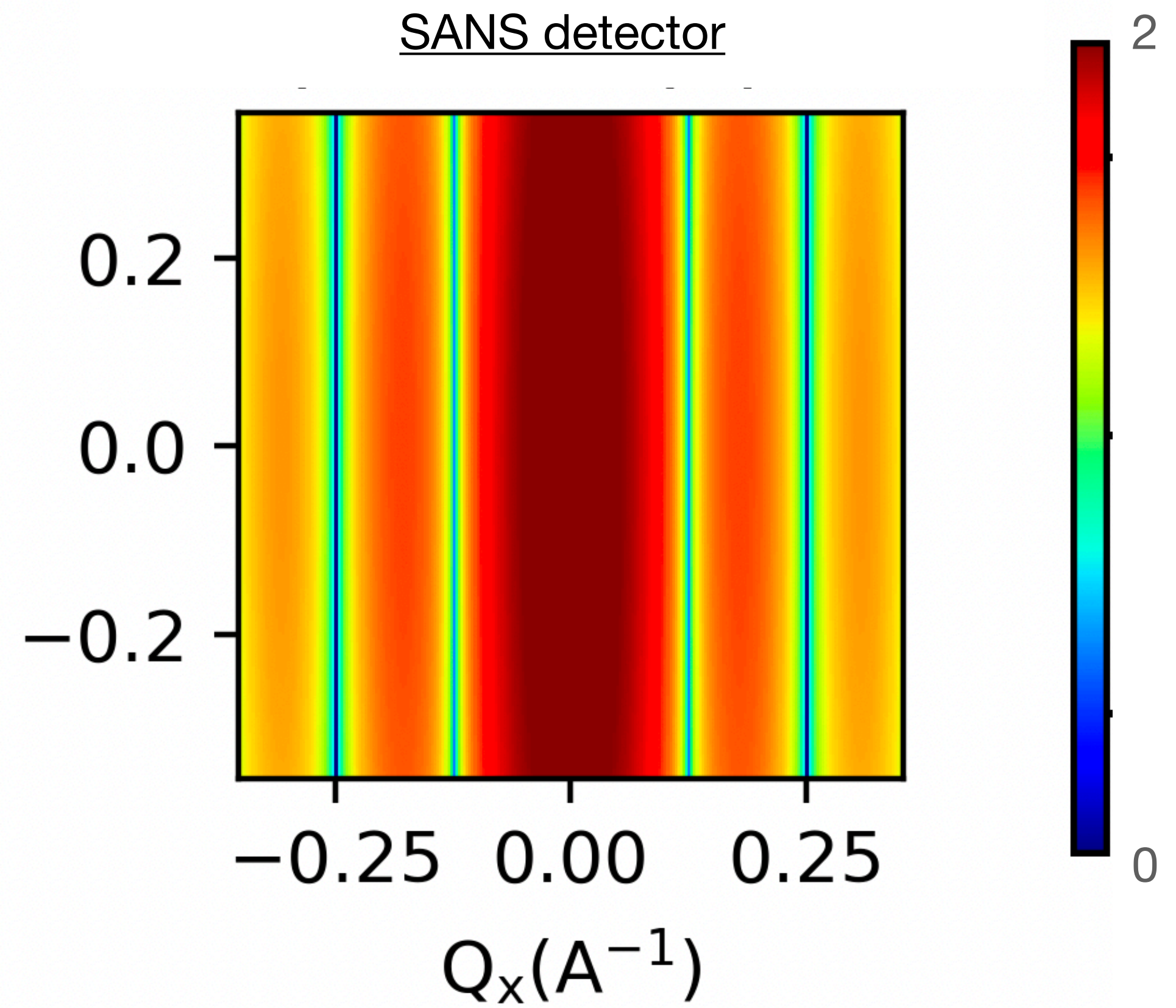
Rule 4: Magnetization perpendicular to the neutron spin only contributes to spin-flip scattering



Example: Unpolarized SANS of a non-magnetic slab



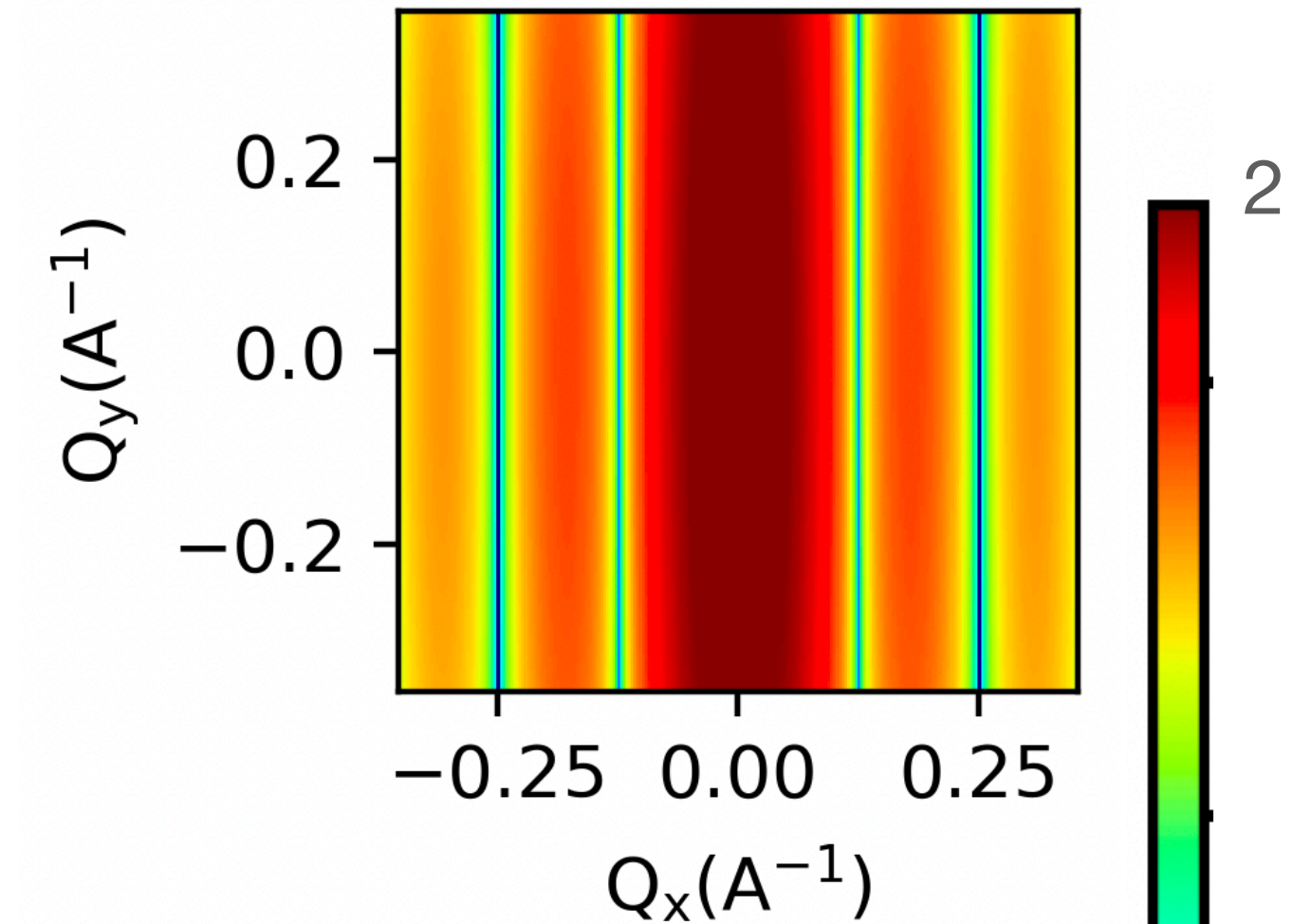
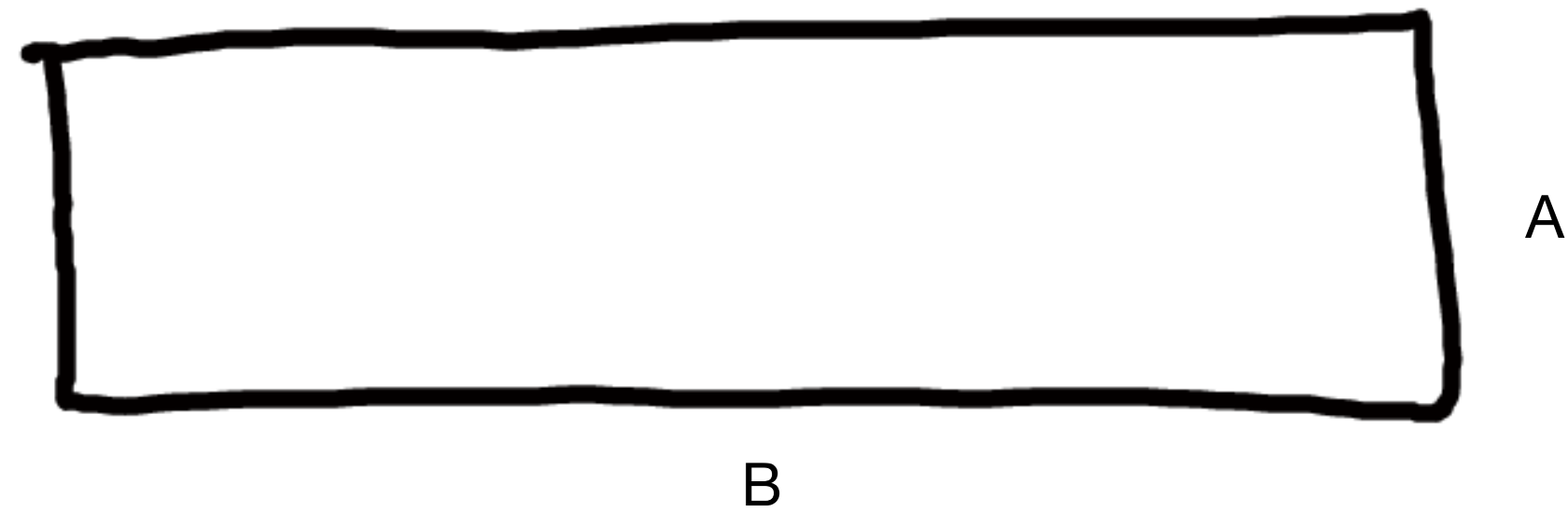
$Q_y(\text{\AA}^{-1})$



Example: Unpolarized SANS of a magnetic slab

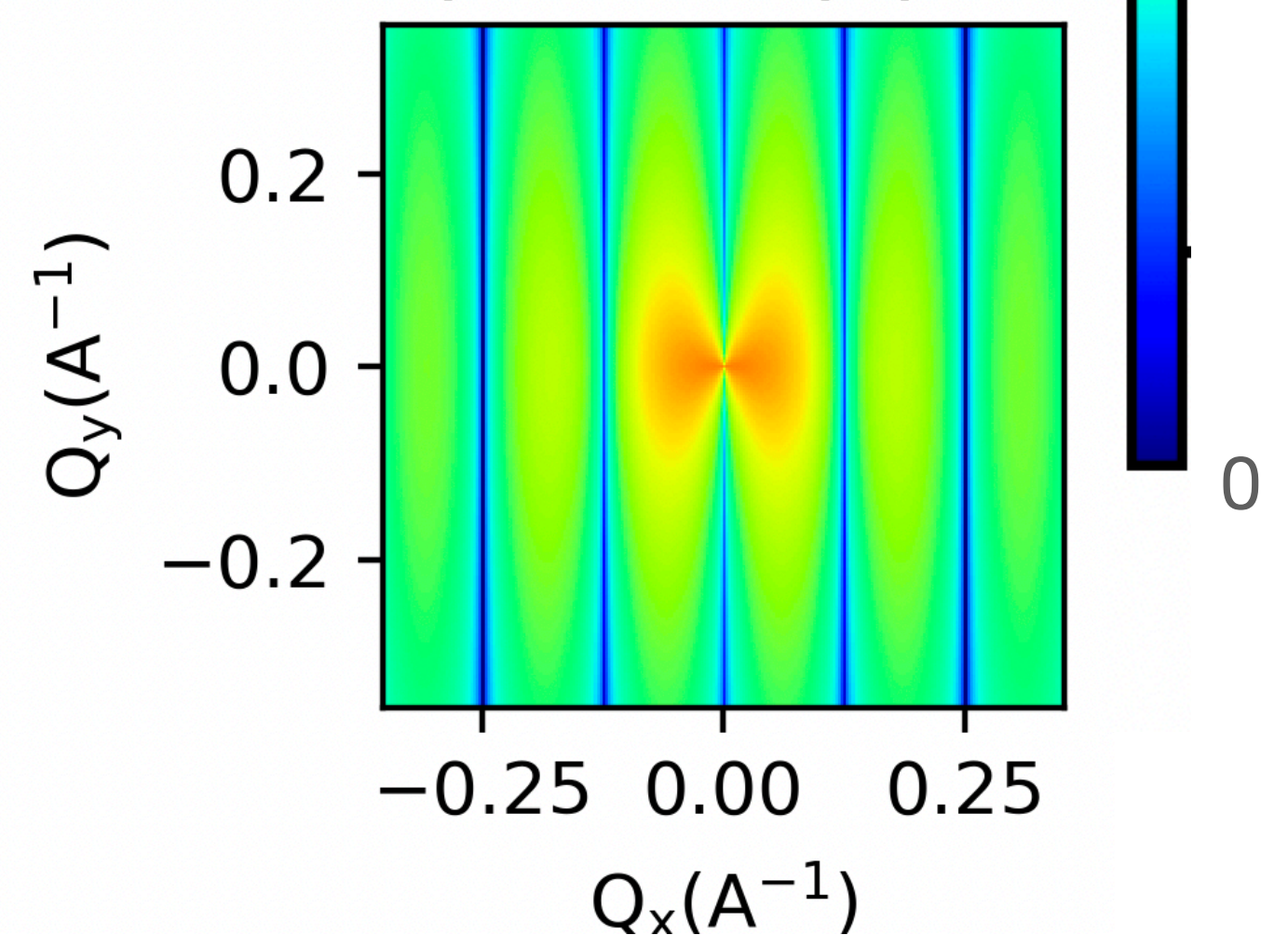
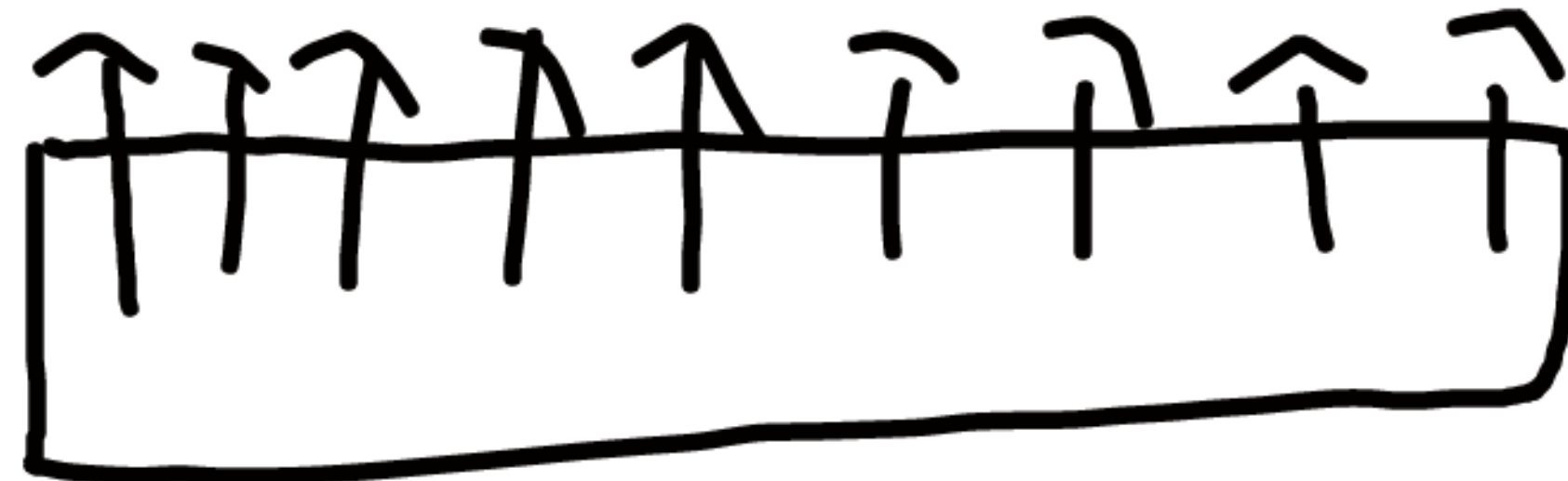
Non-magnetic slab in the beam
(neutron beam goes in the page)

$B \gg A$



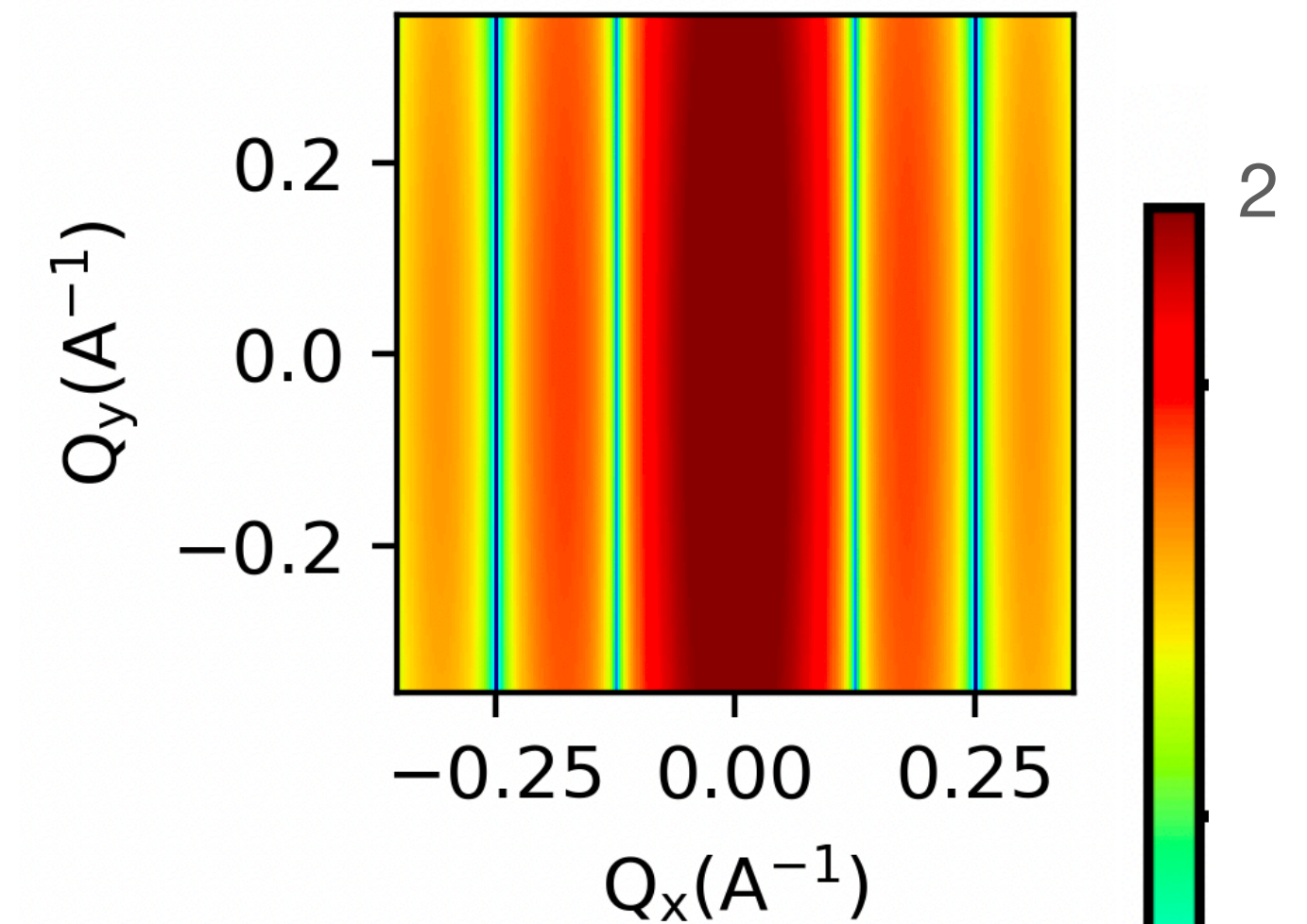
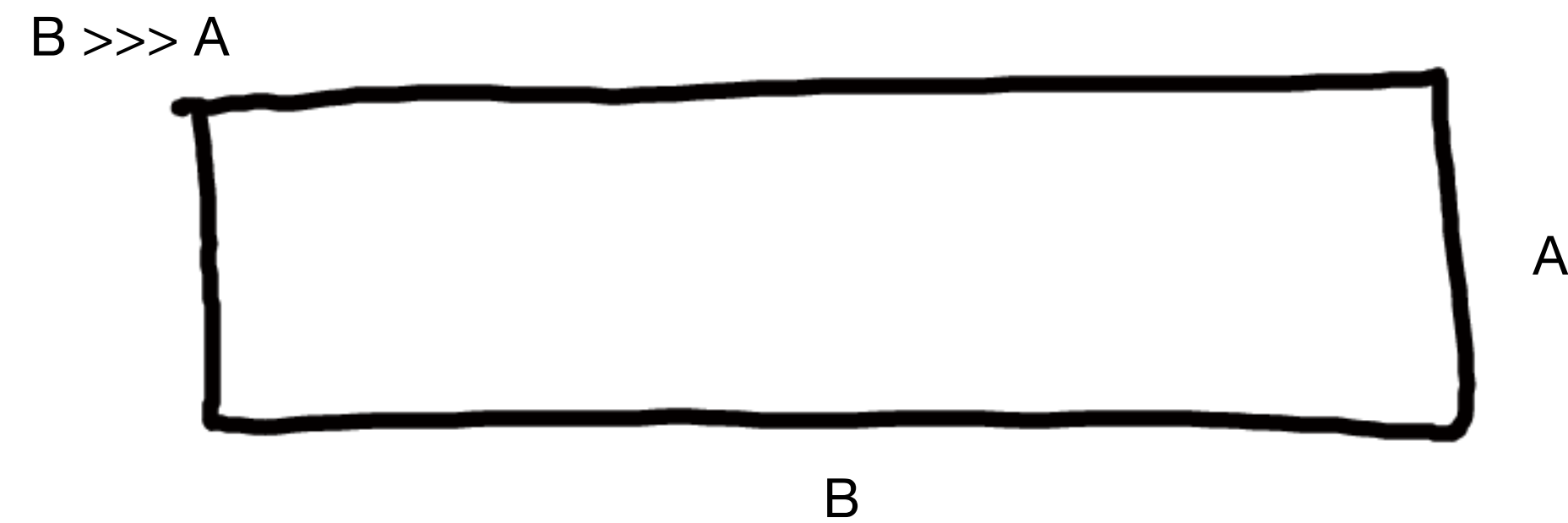
Magnetic slab in the beam
(neutron beam goes in the page)

$M \parallel y$

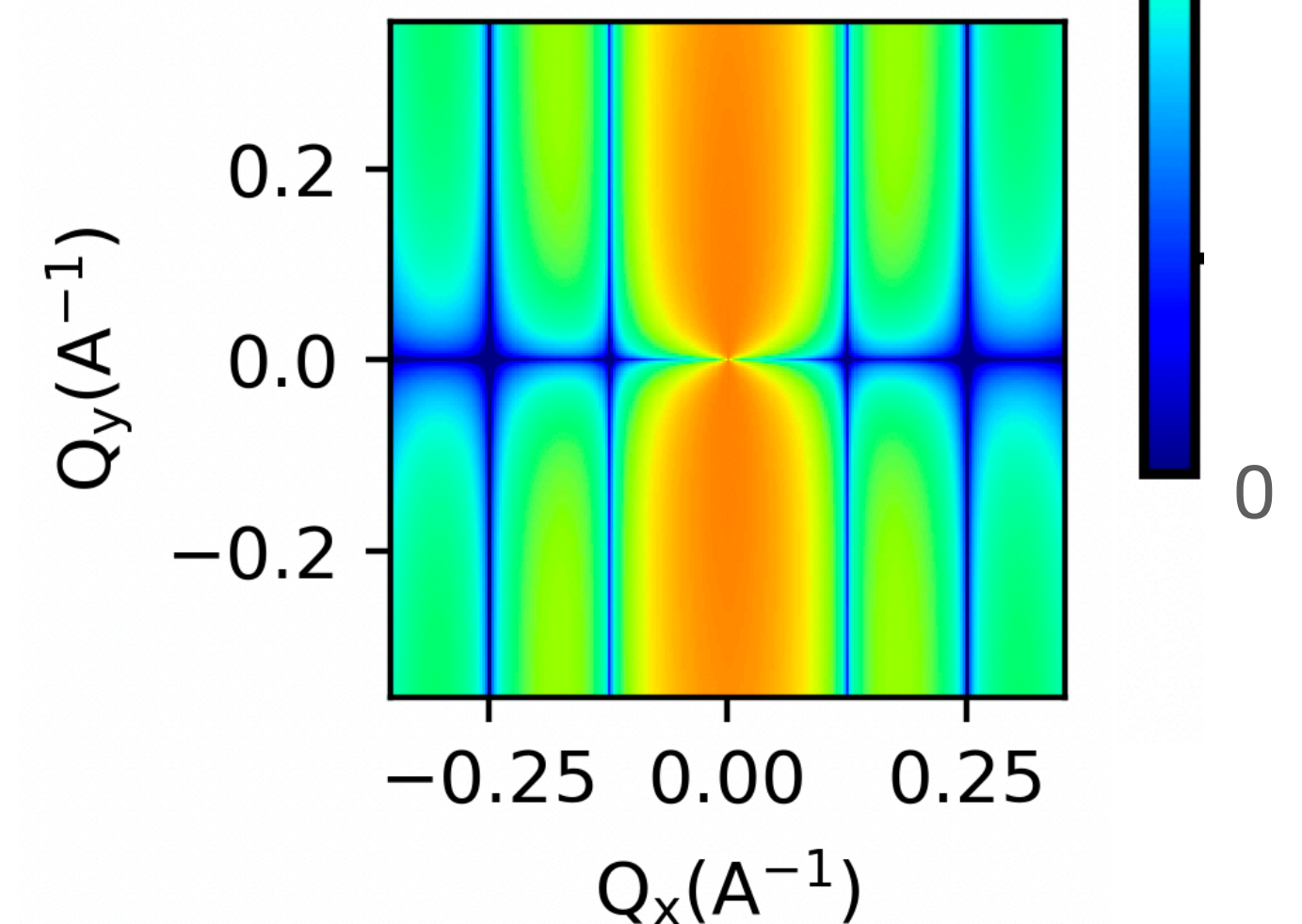


Example: Unpolarized SANS of a magnetic slab

Non-magnetic slab in the beam
(neutron beam goes in the page)

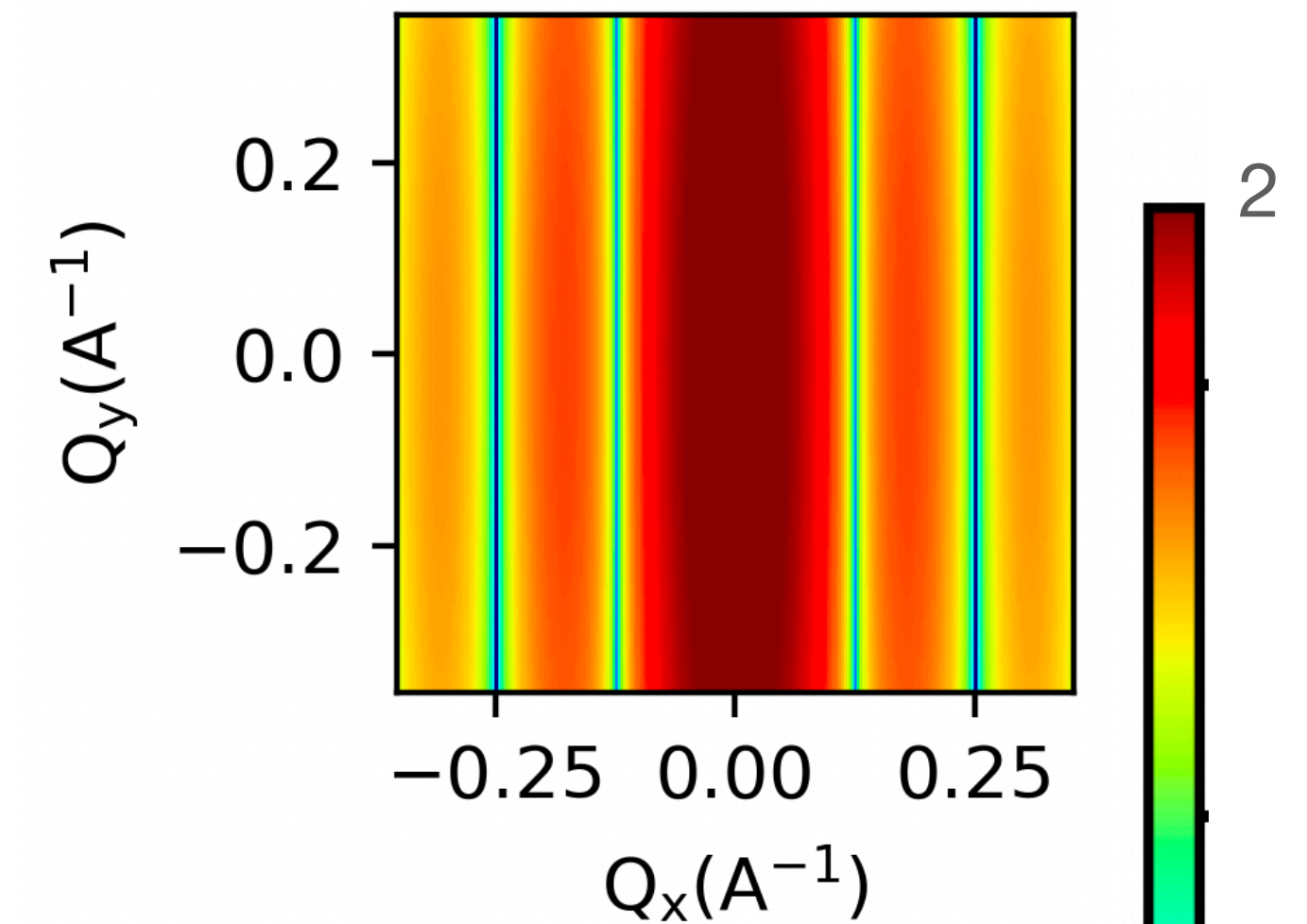
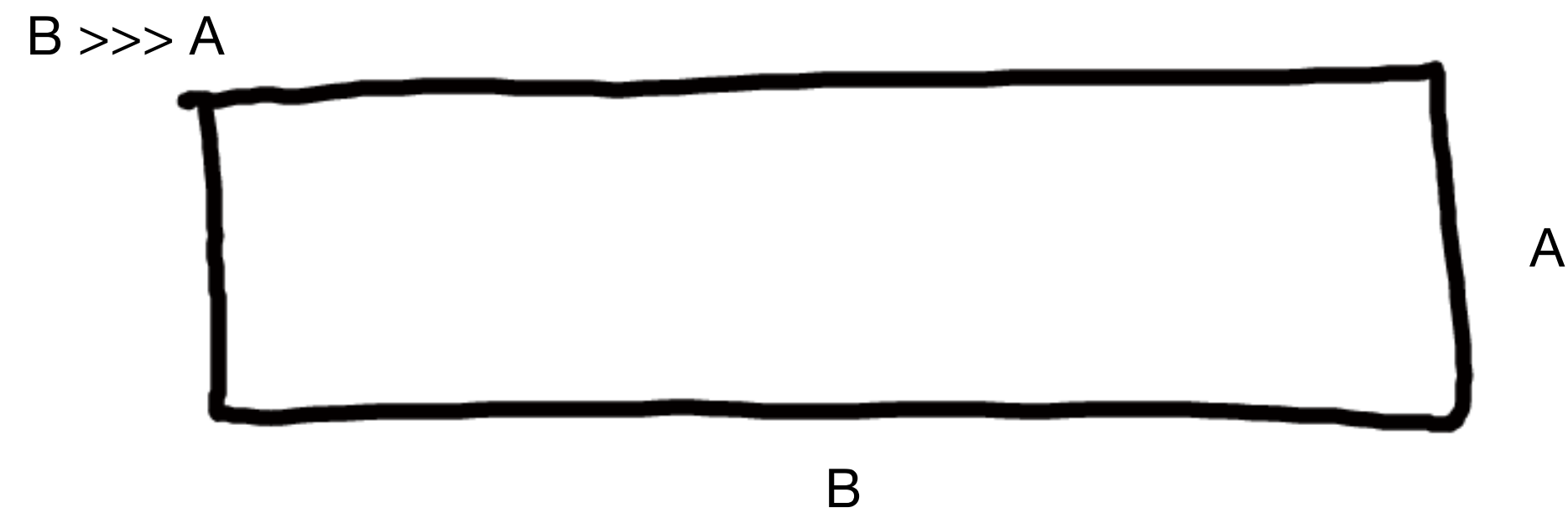


Magnetic slab in the beam
(neutron beam goes in the page)

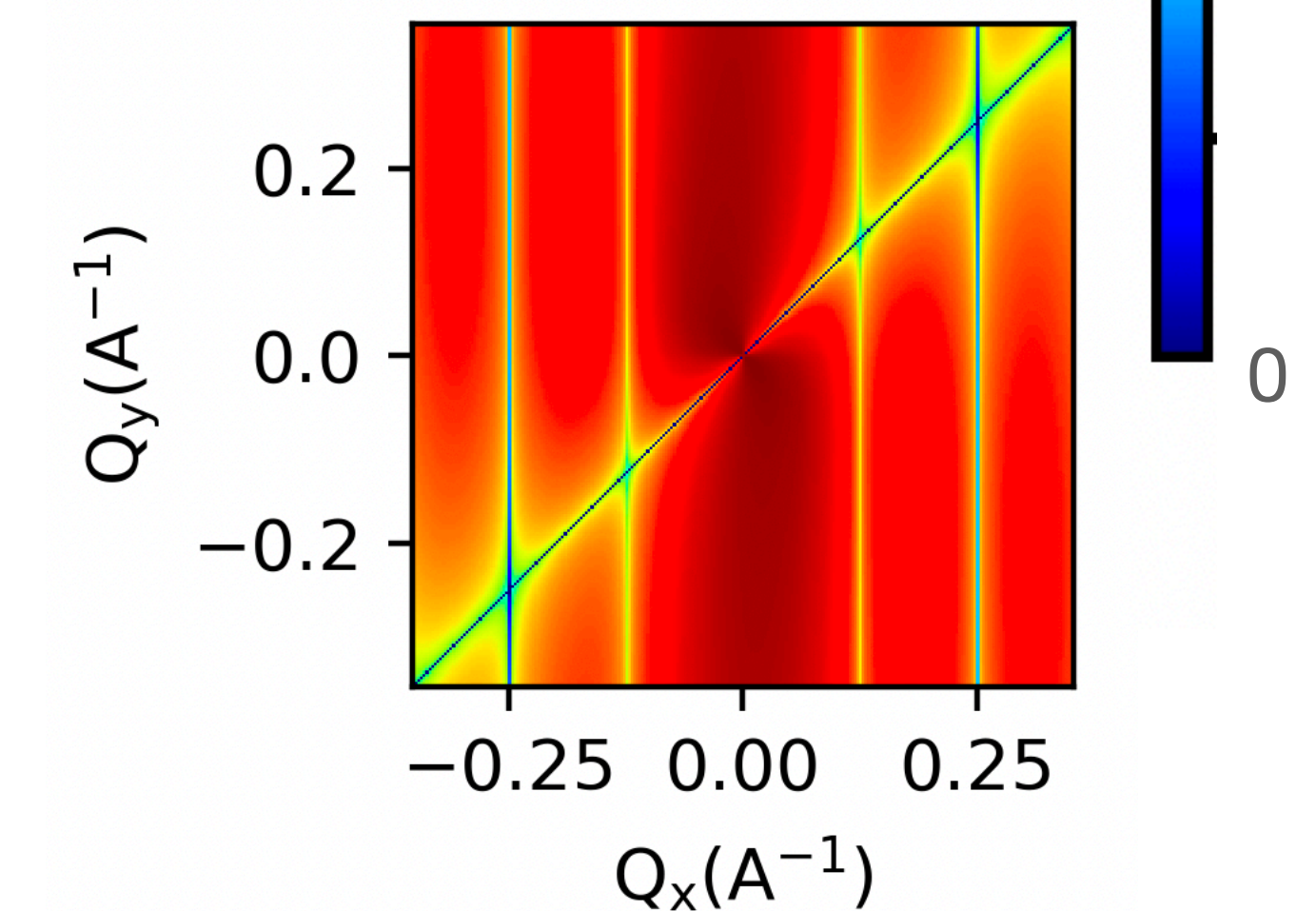
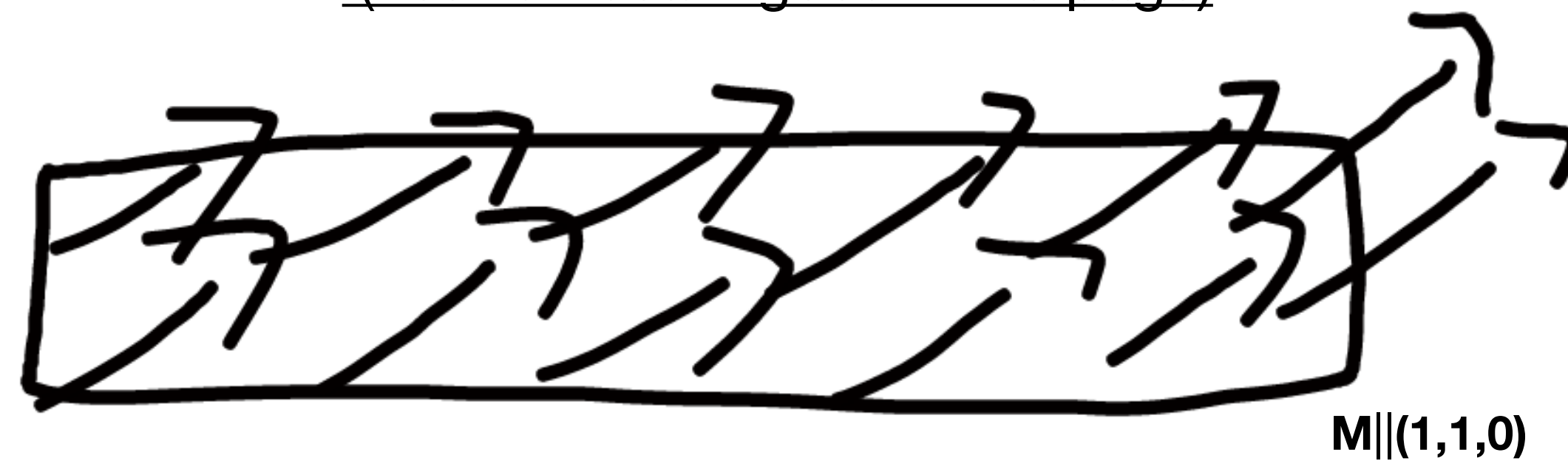


Example: Unpolarized SANS of a magnetic slab

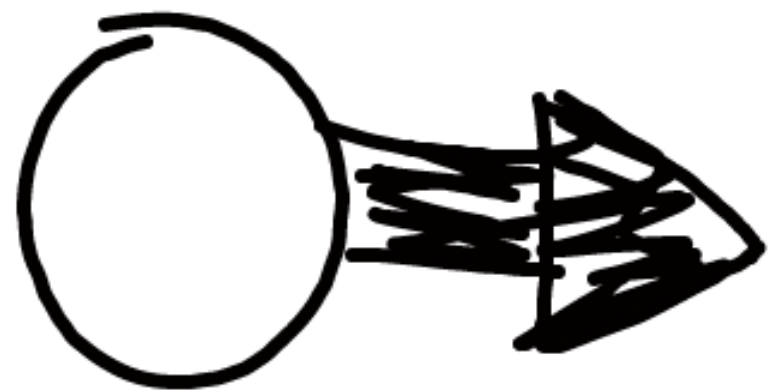
Non-magnetic slab in the beam
(neutron beam goes in the page)



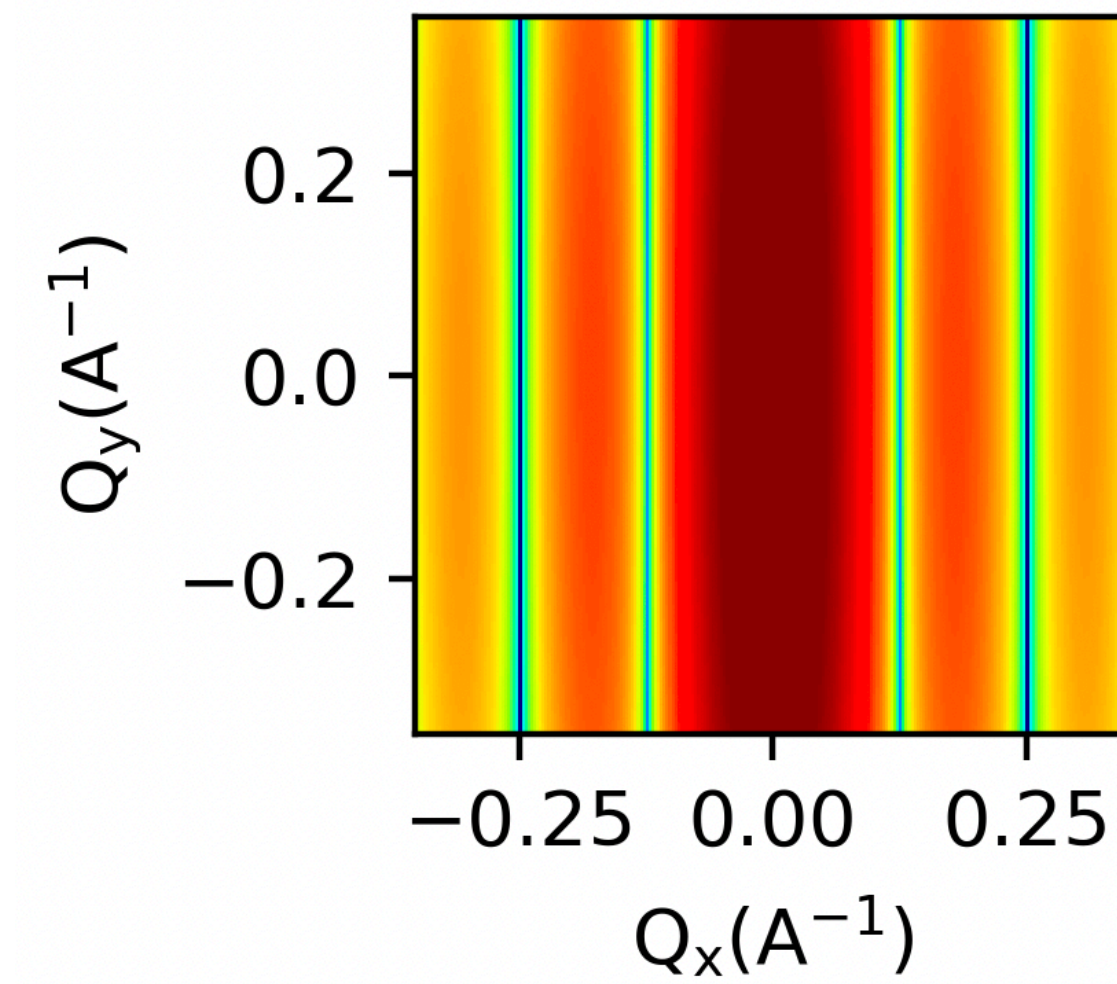
Magnetic slab in the beam
(neutron beam goes in the page)



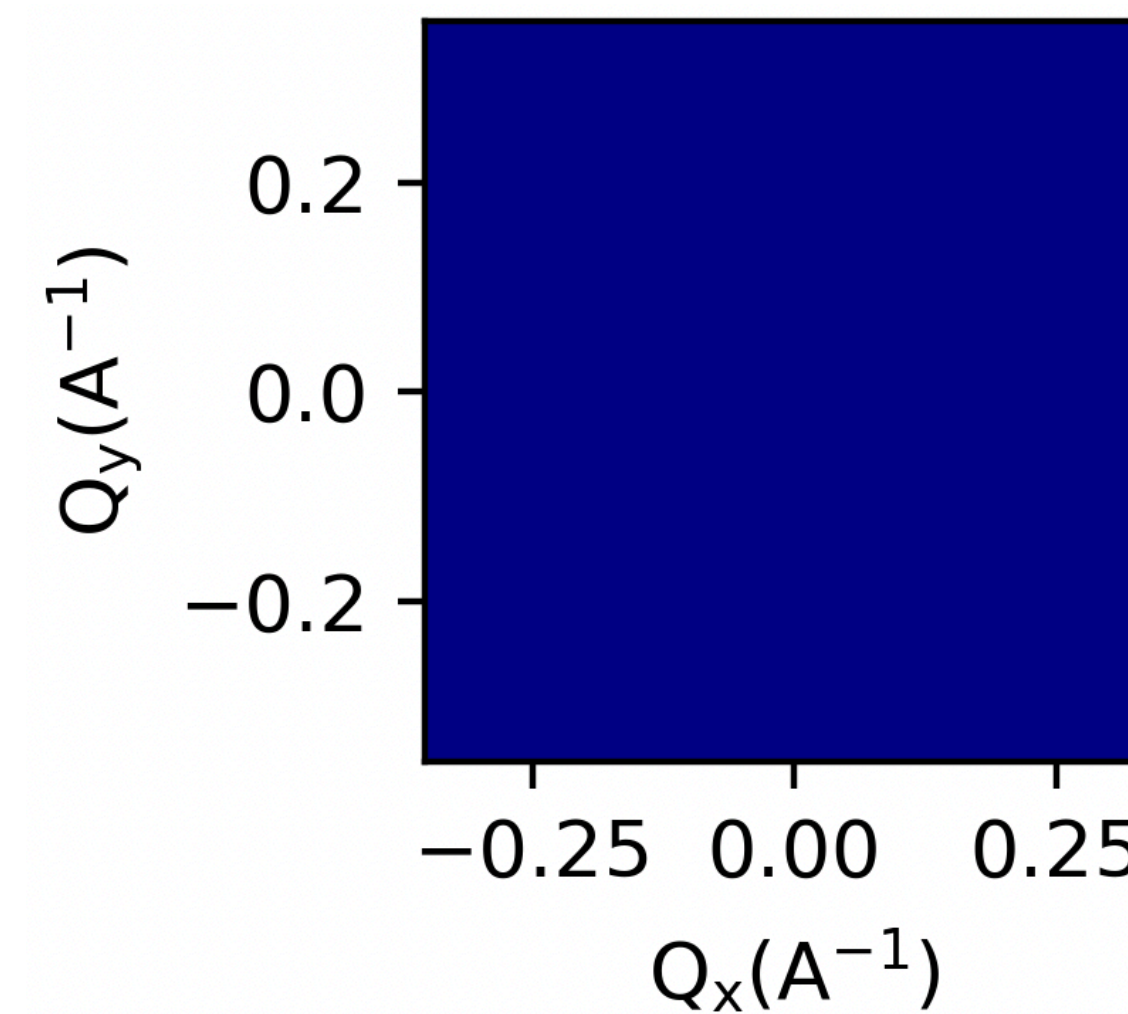
Example: Unpolarized SANS of a magnetic slab



Non Spin-Flip (NSF)



Spin-Flip (SF)



Uniaxial Polarization Analysis : General Rules

- 1) Nuclear scattering is always non-spin-flip (NSF)
- 2) The magnetization parallel to the scattering momentum vector $\mathbf{Q}$ cannot be observed.
- 3) and 4) Magnetic scattering is both spin-flip (SF) and NSF. The NSF is given by the magnetization *parallel* to the neutron spin, while the SF is given by the magnetization *perpendicular* to the neutron spin.

Nuclear Scattering

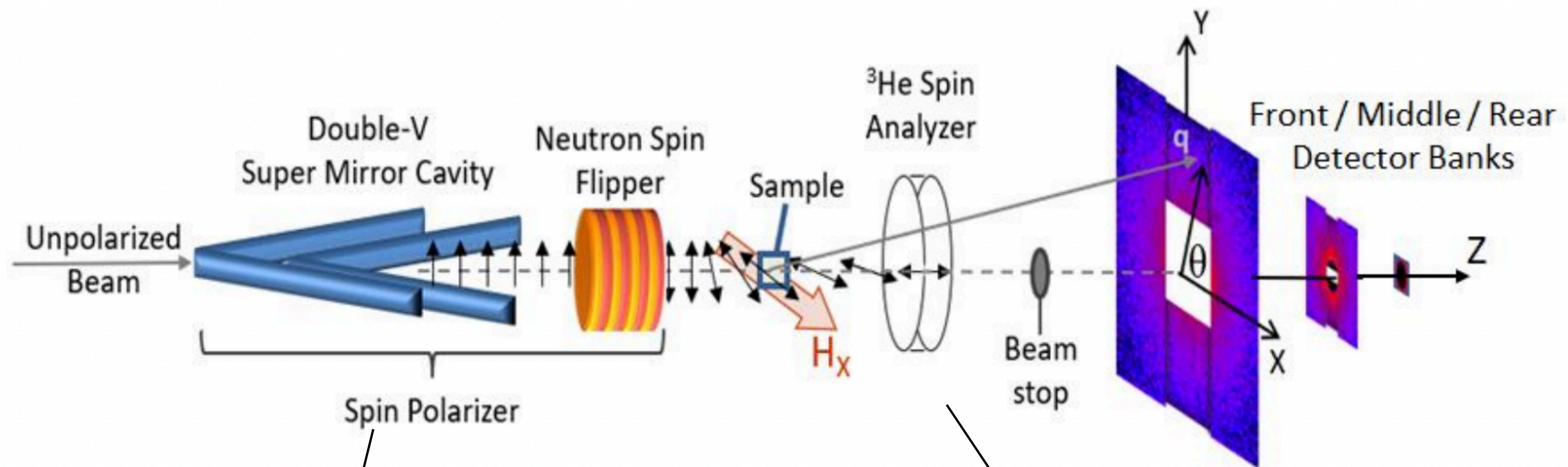
$$\langle \mathbf{S}' | b | \mathbf{S} \rangle = b \langle \mathbf{S}' | \mathbf{S} \rangle = \begin{cases} b \begin{cases} |\uparrow\rangle \rightarrow |\uparrow\rangle \\ |\downarrow\rangle \rightarrow |\downarrow\rangle \end{cases} & \text{Non-spin-flip} \\ 0 \begin{cases} |\uparrow\rangle \rightarrow |\downarrow\rangle \\ |\downarrow\rangle \rightarrow |\uparrow\rangle \end{cases} & \text{Spin-flip} \end{cases}$$

Magnetic Scattering

$$\langle \mathbf{S}' | V_m(\mathbf{Q}) | \mathbf{S} \rangle = -\frac{\gamma_n r_0}{2\mu_B} \begin{cases} M_{\perp z}(\mathbf{Q}) & |\uparrow\rangle \rightarrow |\uparrow\rangle \\ -M_{\perp z}(\mathbf{Q}) & |\downarrow\rangle \rightarrow |\downarrow\rangle \\ M_{\perp x}(\mathbf{Q}) - iM_{\perp y}(\mathbf{Q}) & |\uparrow\rangle \rightarrow |\downarrow\rangle \\ M_{\perp x}(\mathbf{Q}) + iM_{\perp y}(\mathbf{Q}) & |\downarrow\rangle \rightarrow |\uparrow\rangle \end{cases}$$

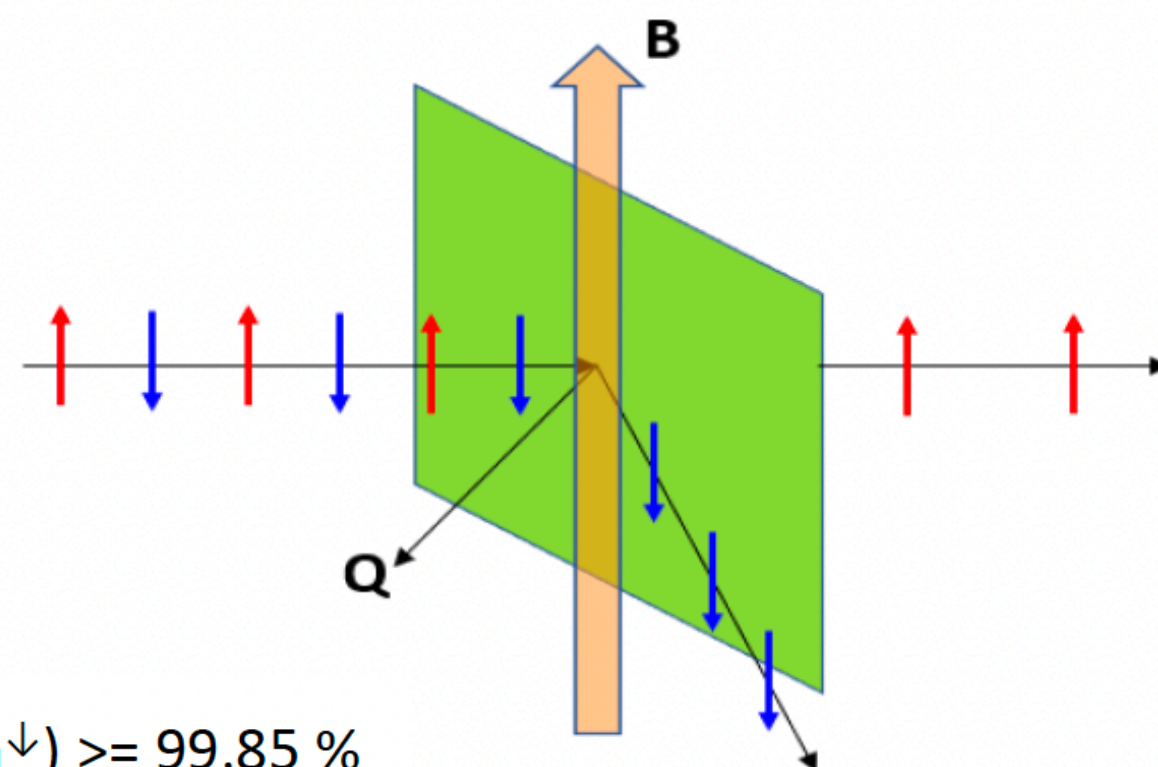
V-SANS : uniaxial polarization analysis can be performed

S. Oberdick et al., Sci. Reports 8, 3425 (2018)

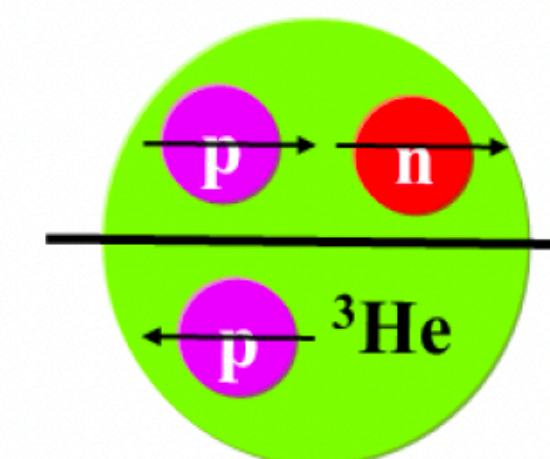


Spin Polarizer

He3 Analyzer



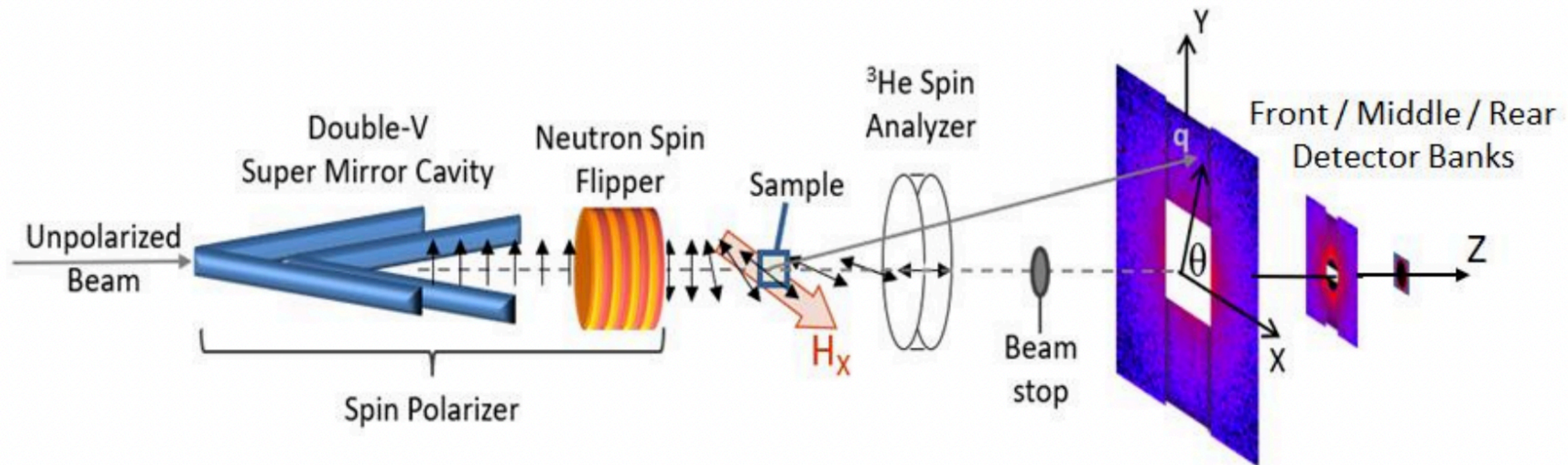
$$P \equiv (n^{\uparrow} - n^{\downarrow}) / (n^{\uparrow} + n^{\downarrow}) \geq 99.85 \%$$



Neutrons aligned
with ^3He pass;
otherwise
absorbed

V-SANS : Uniaxial polarization analysis can be performed

S. Oberdick et al., Sci. Reports 8, 3425 (2018)



1) Unpolarized

$$I(q) = I_{uu}(q) + I_{ud}(q) + I_{du}(q) + I_{dd}(q)$$

2) Half-polarized (polarizer only)

$$I_u(q) = I_{uu}(q) + I_{ud}(q)$$

$$I_d(q) = I_{du}(q) + I_{dd}(q)$$

3) Full polarized mode (polarizer and analyzer)

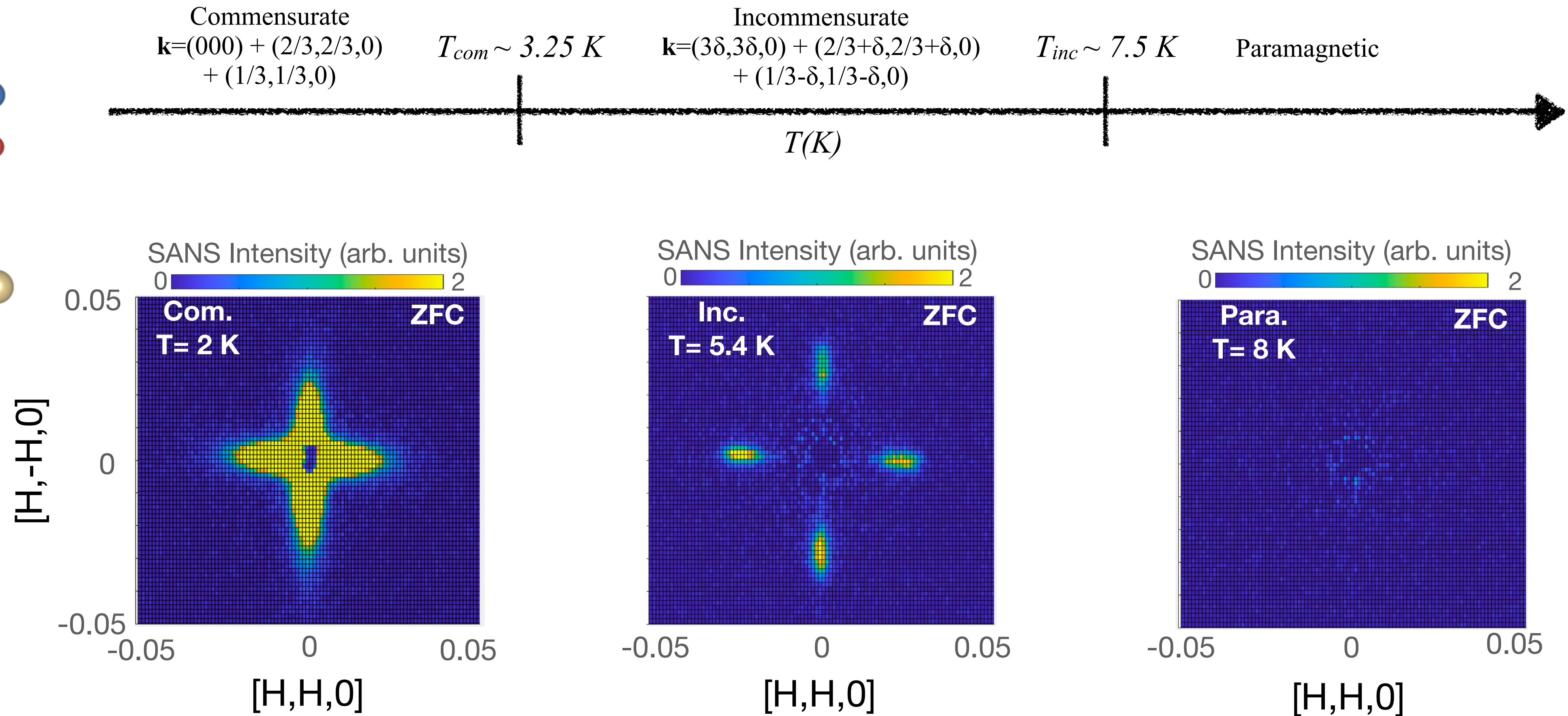
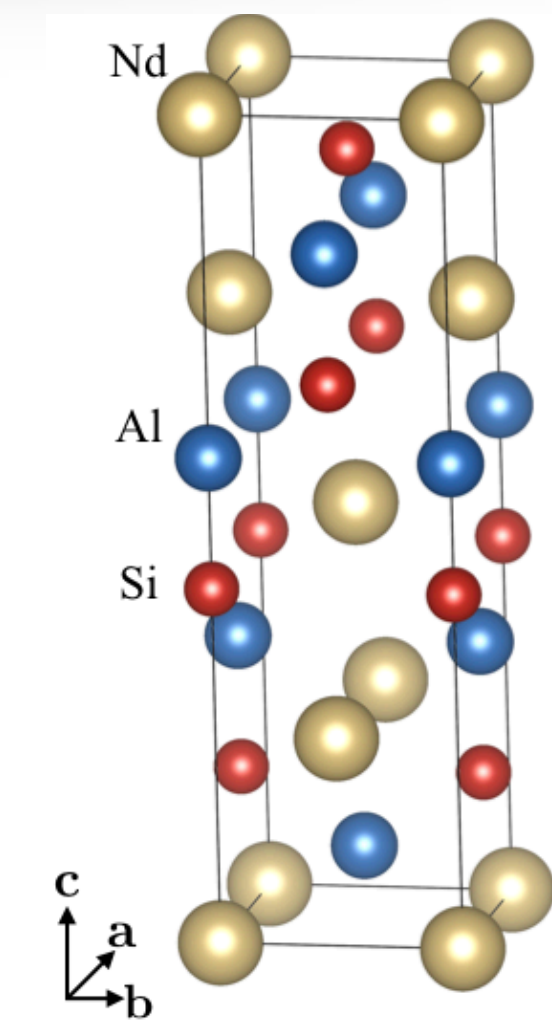
$$I_{uu}(q)$$

$$I_{ud}(q)$$

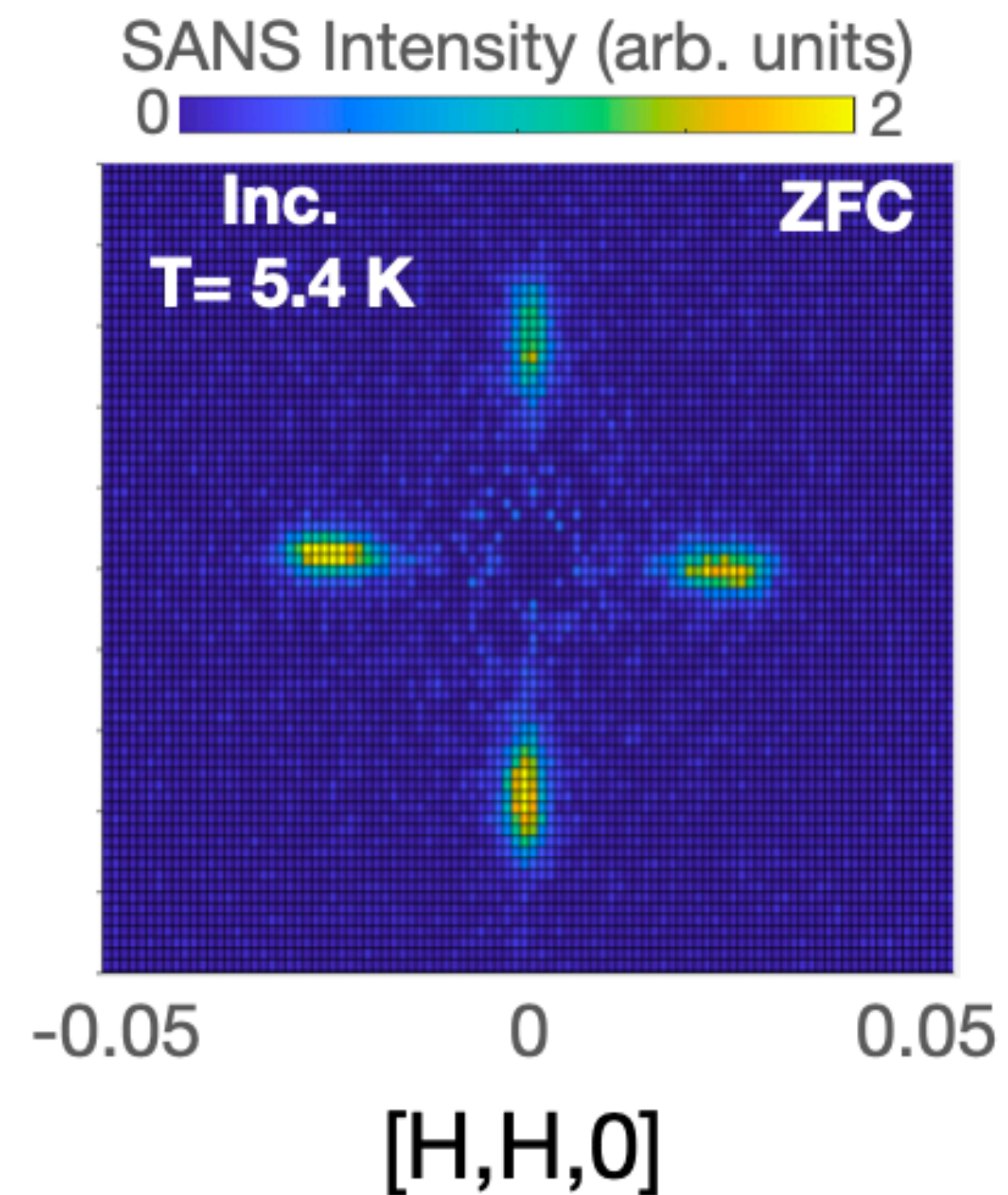
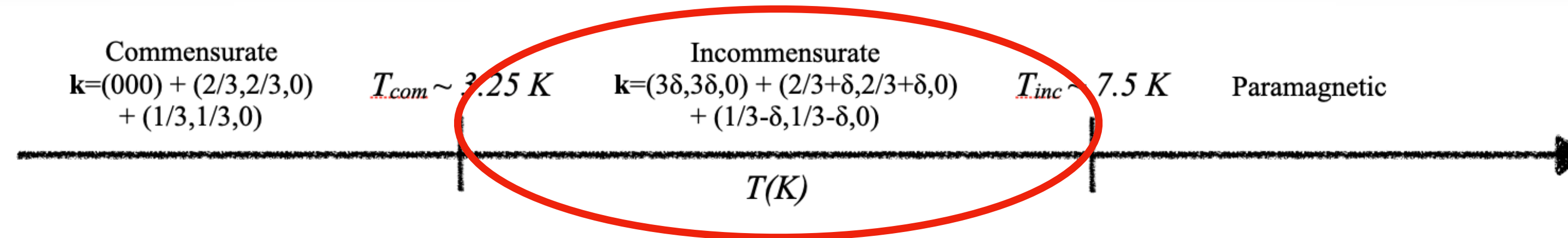
$$I_{du}(q)$$

$$I_{dd}(q)$$

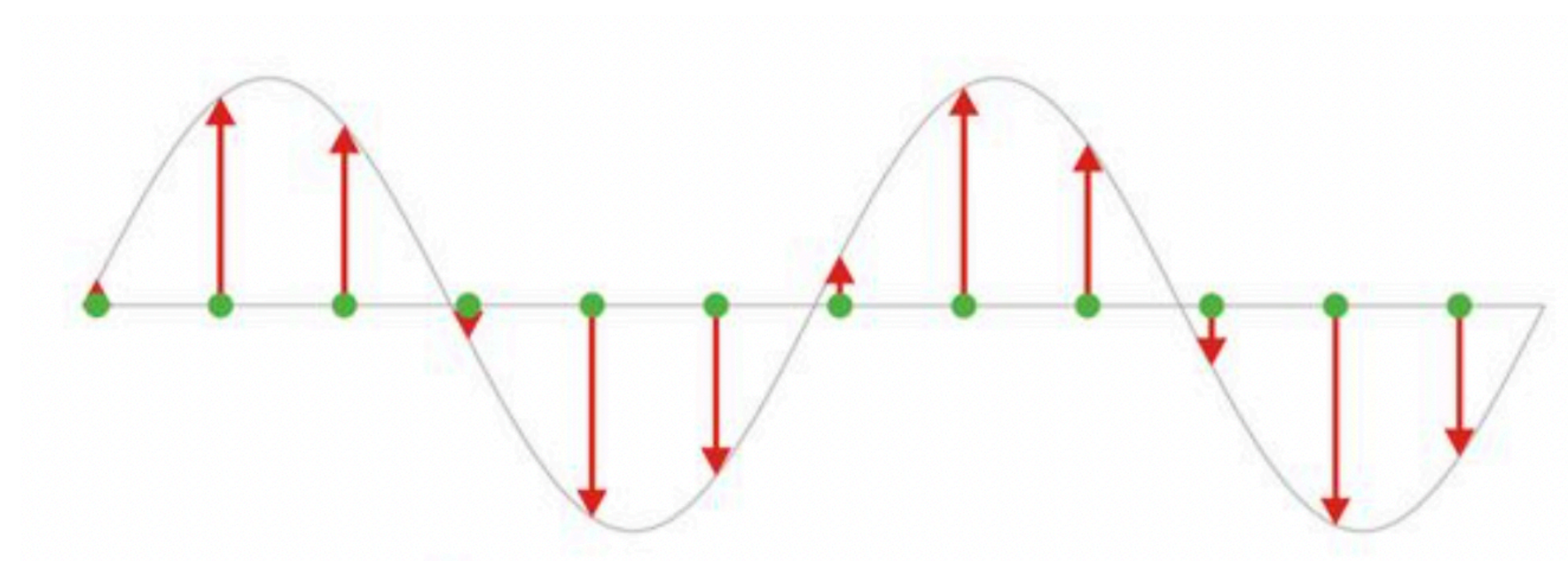
Unpolarized SANS: Weyl-mediated magnetism in NdAlSi



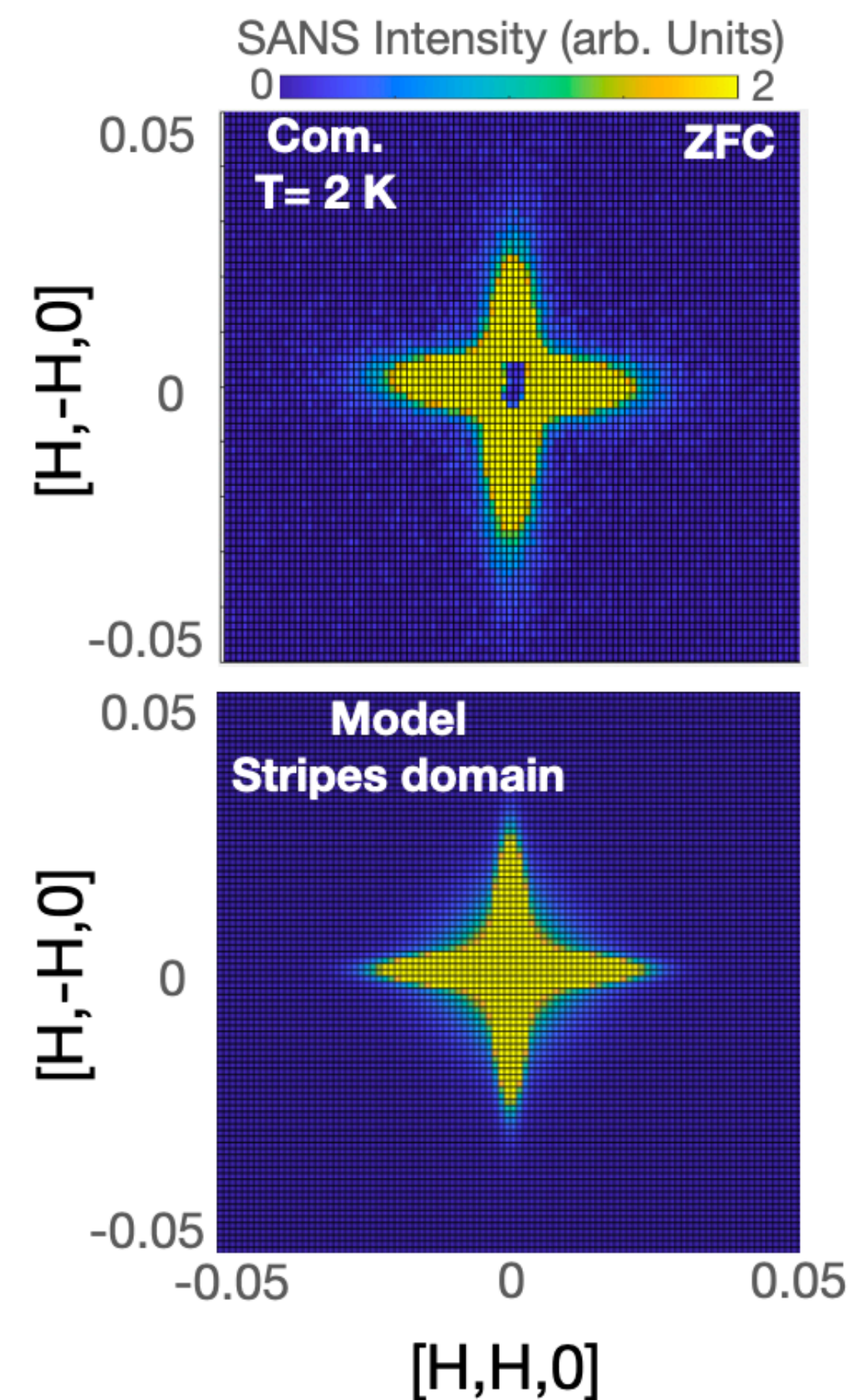
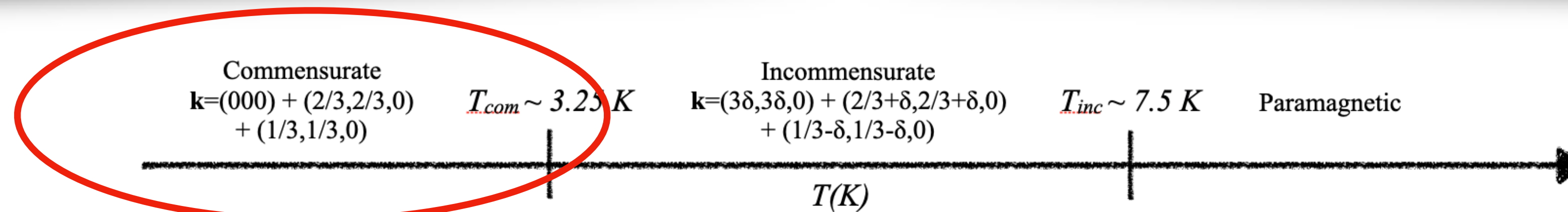
Unpolarized SANS: Weyl-mediated magnetism in NdAlSi



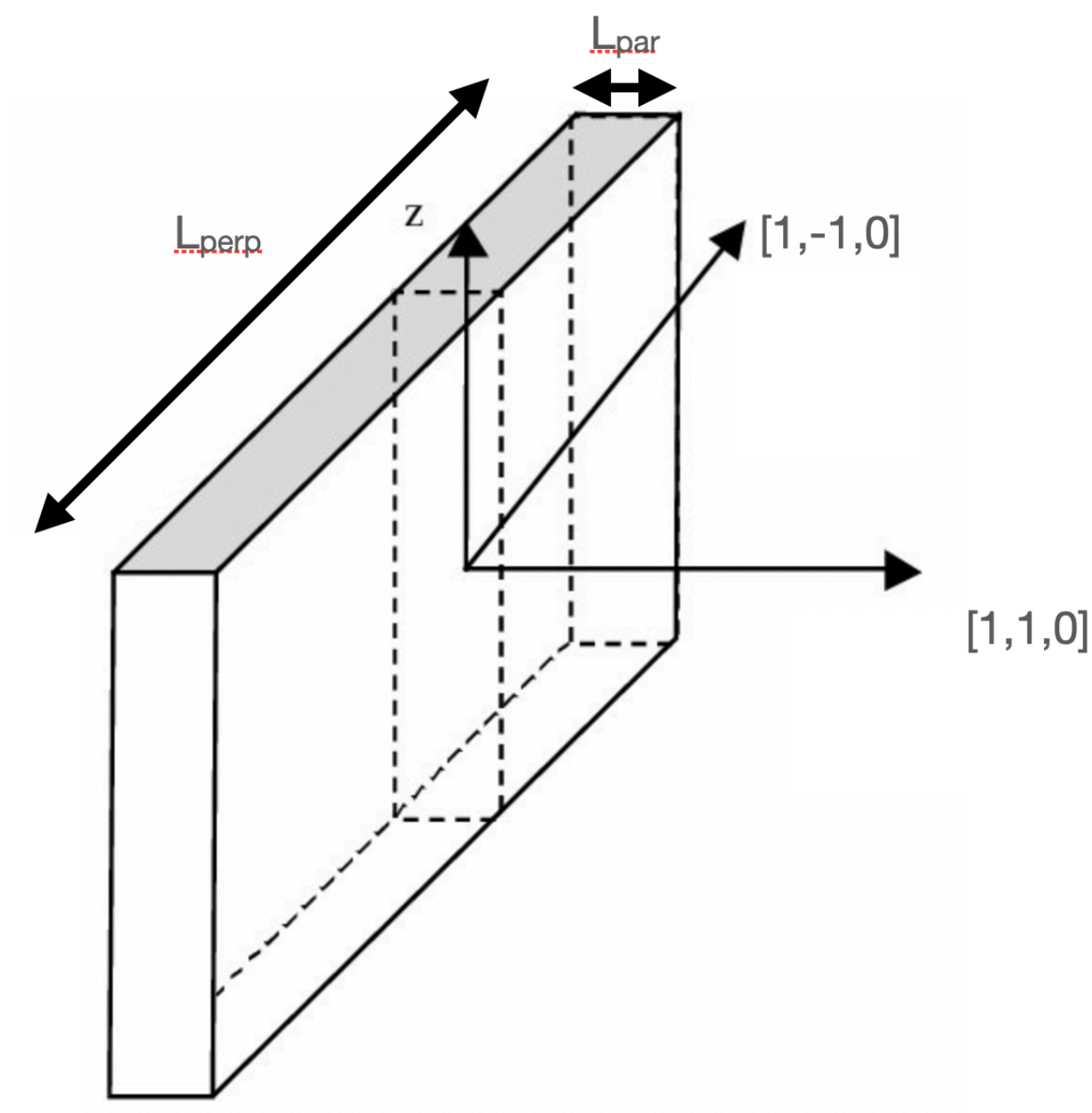
Modulated spin density wave



Unpolarized SANS: Weyl-mediated magnetism in NdAlSi

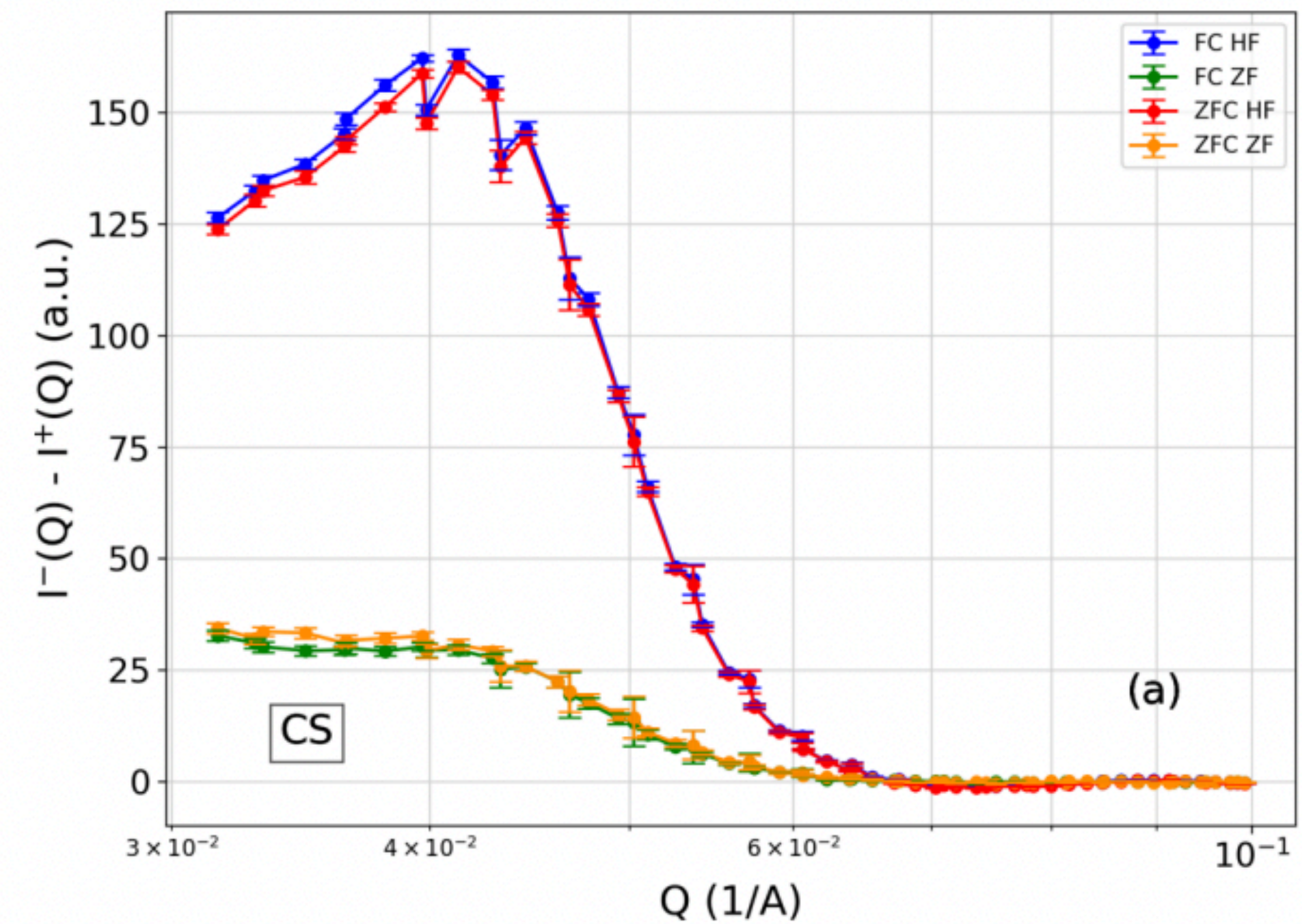
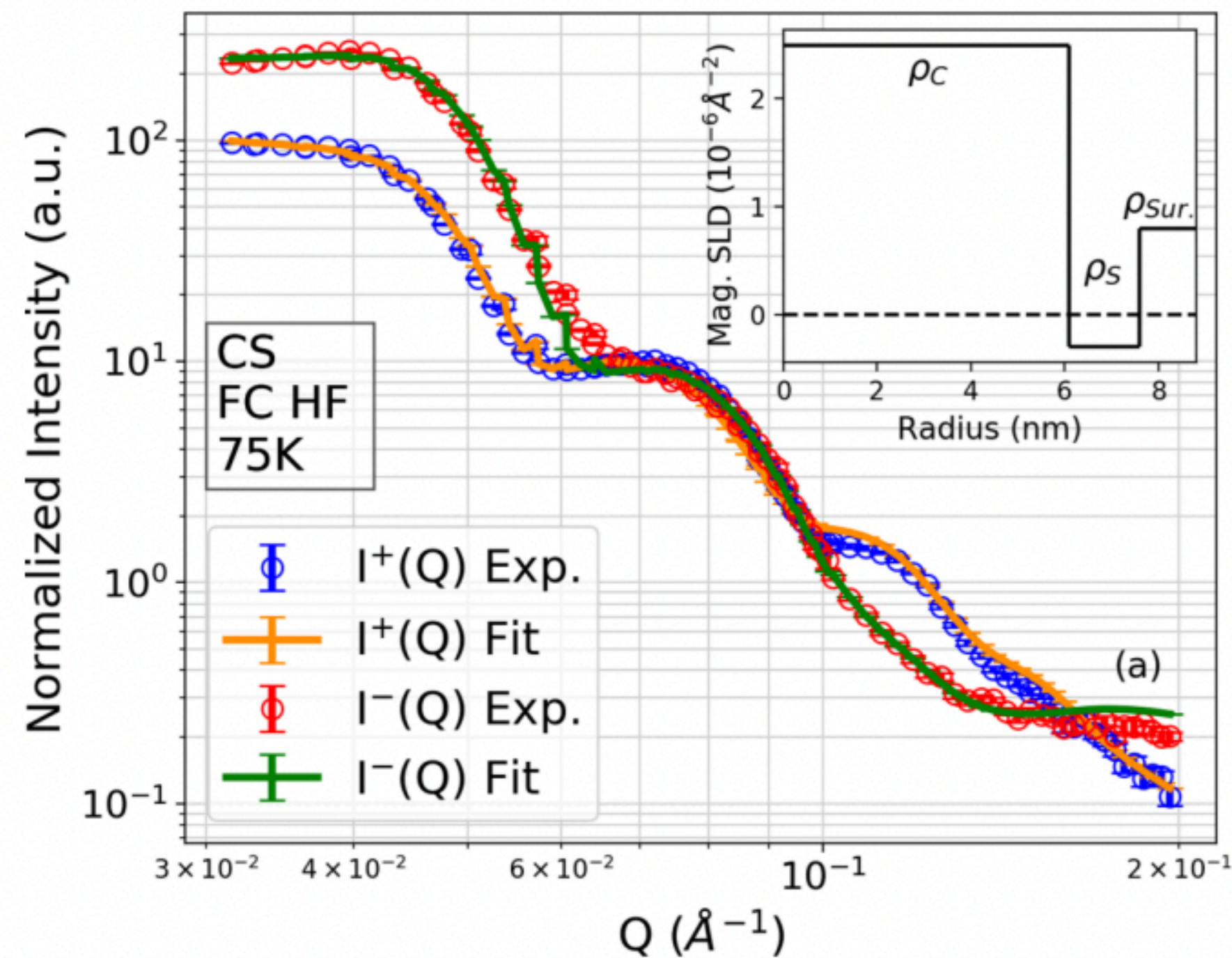
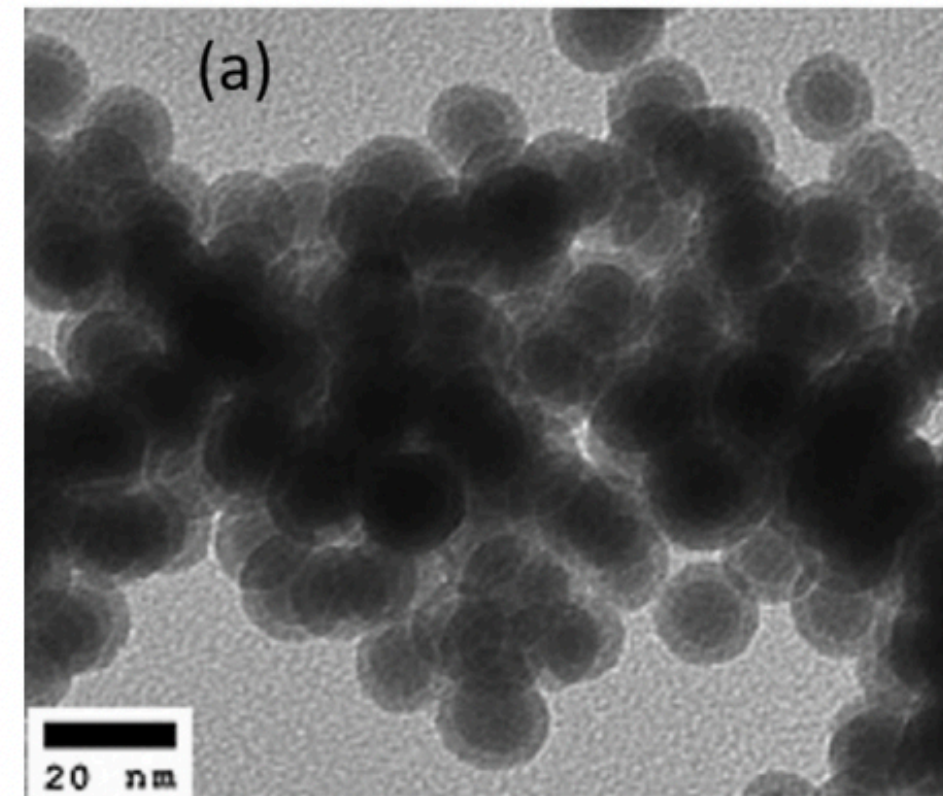


Stripe ferromagnetic domains

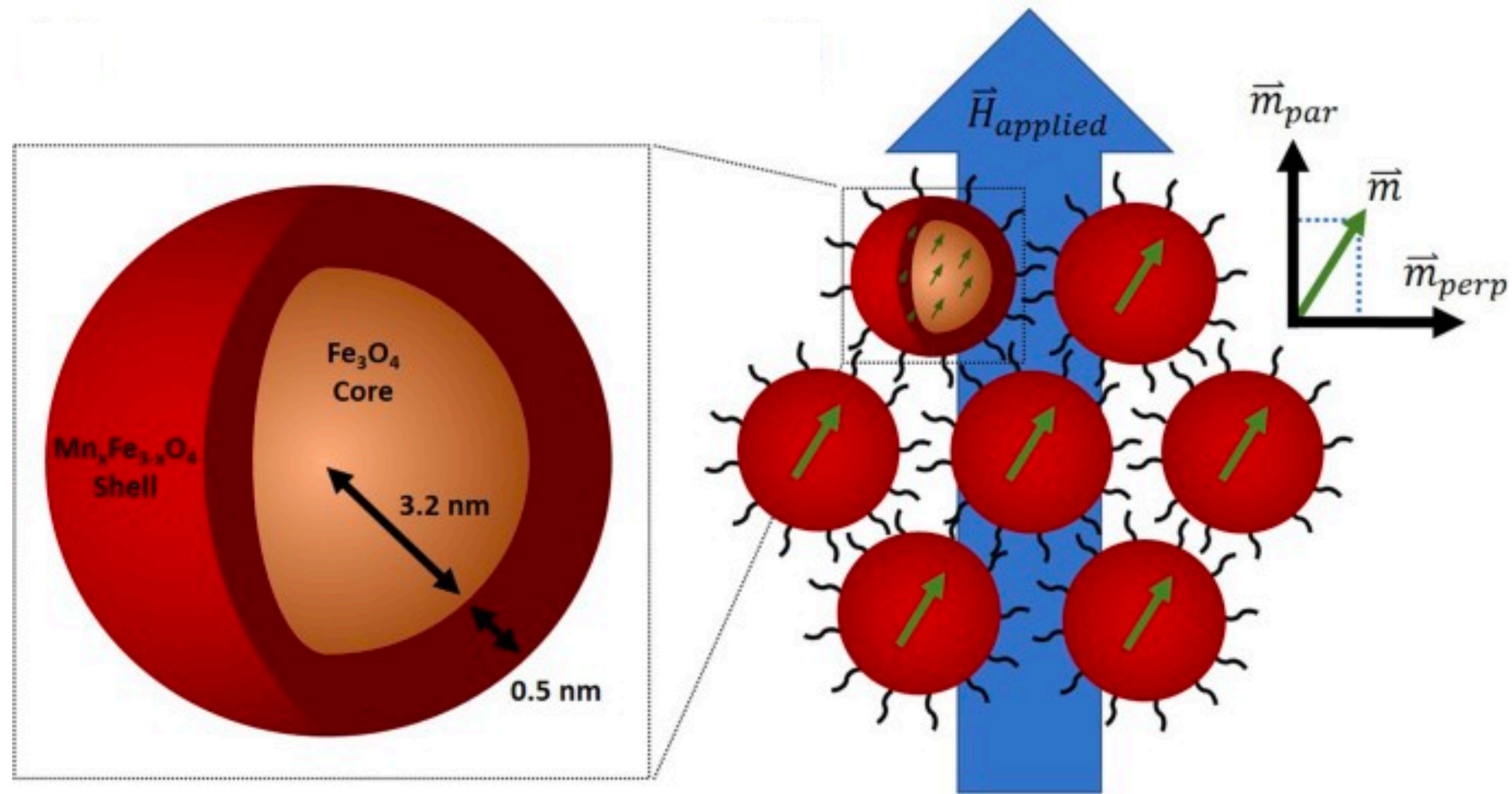


Example of Half-polarized SANS: Identifying the weak magnetic scattering in nanoparticles (NPs) system

C. Kons et al., PRM 4, 034408 (2020)

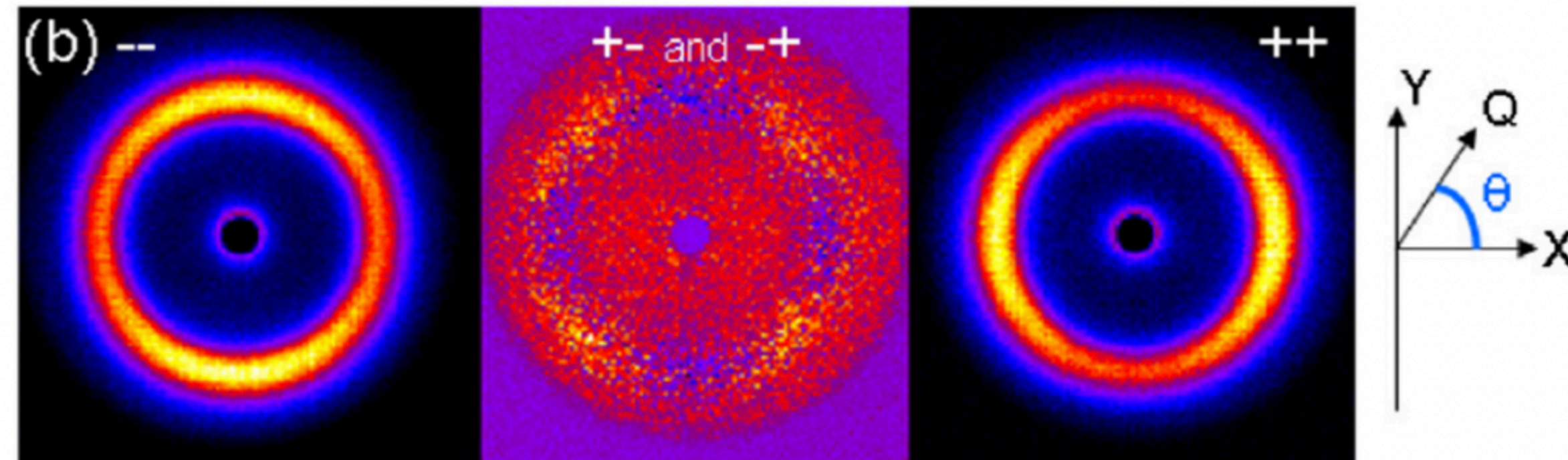
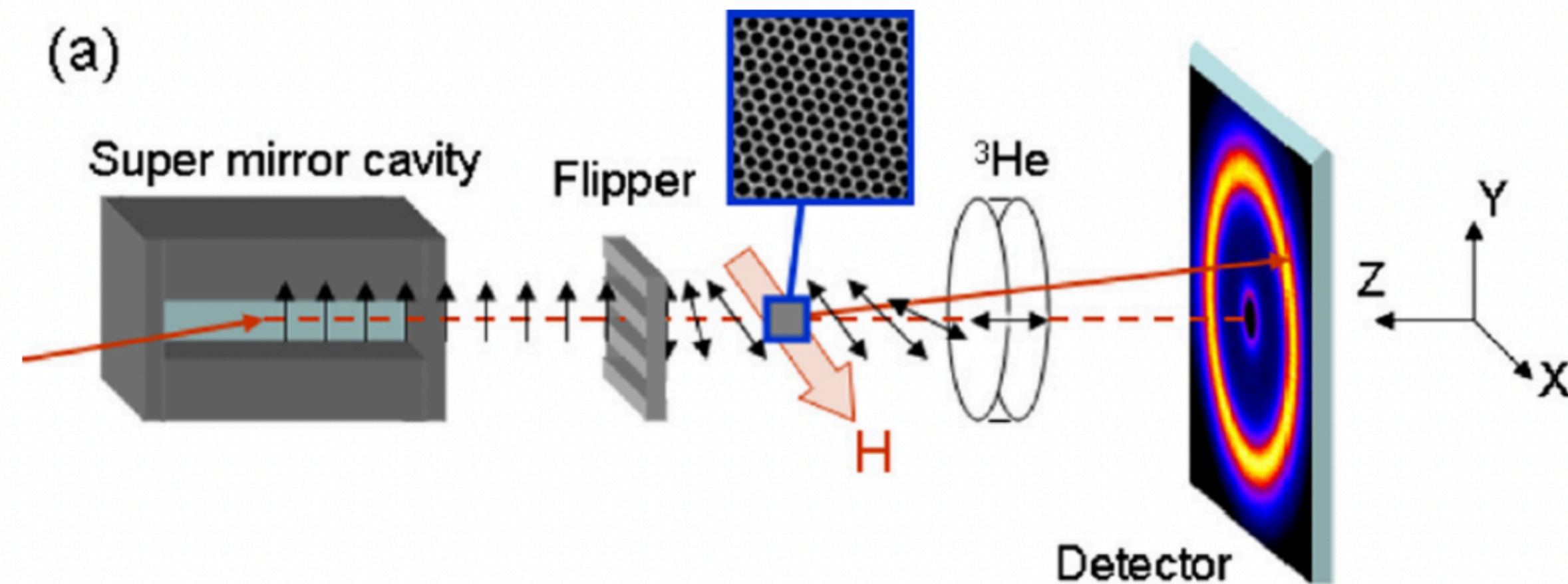


Full-polarized SANS: Characterizing the in-field 3D magnetization profile of NP's



S. Oberdick et al., Sci. Reports 8, 3425 (2018)

Full-polarized SANS: Characterizing the in-field 3D magnetization profile of NP's



$$I_{\phi=0^\circ}^{++,--} = N^2,$$

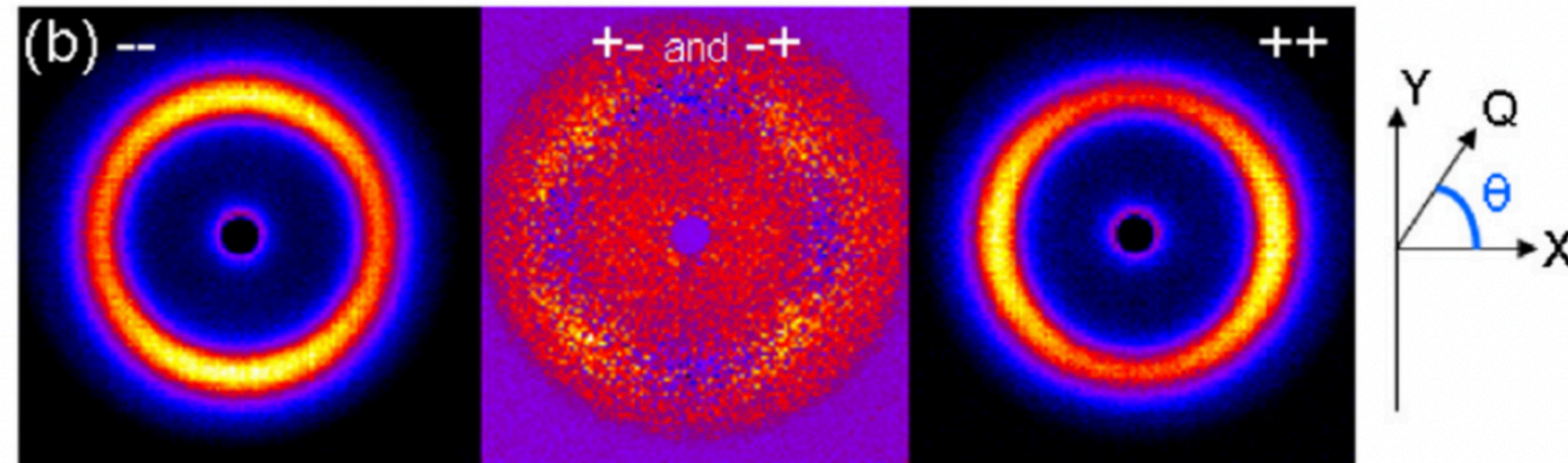
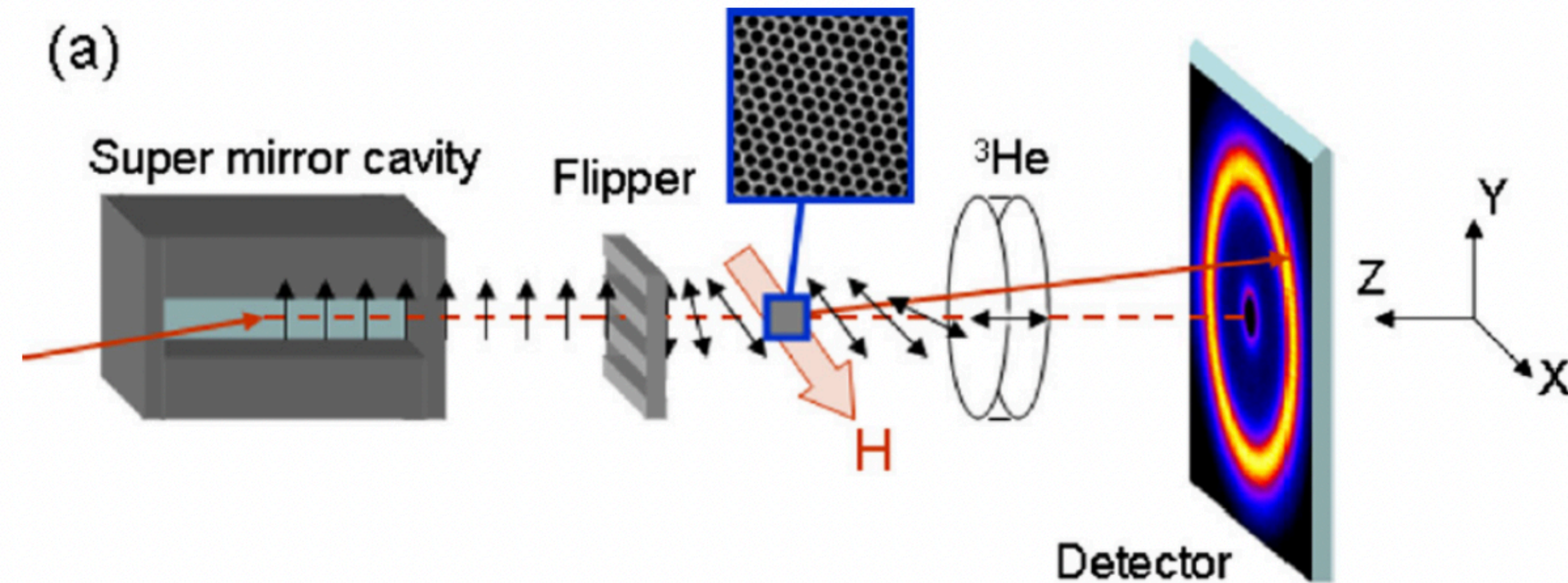
$$I_{\phi=90^\circ}^{++,--} = N^2 + M_X^2 \mp 2NM_X,$$

$$I_{\phi=0^\circ}^{+-,-+} = M_Y^2 + M_Z^2 = 2M_{\text{PERP}}^2,$$

$$I_{\phi=90^\circ}^{+-,-+} = M_Z^2 = M_{\text{PERP}}^2,$$

K. L. Krycka, et al., J Appl Cryst **45**, 554–565 (2012)

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$$N^2(Q) = \frac{1}{2}(I_{\theta=0^\circ}^{++} + I_{\theta=0^\circ}^{--}),$$

$$M_{\text{PARL}}^2(Q) = \frac{(I_{\theta=90^\circ}^{--} - I_{\theta=90^\circ}^{++})^2}{16N^2},$$

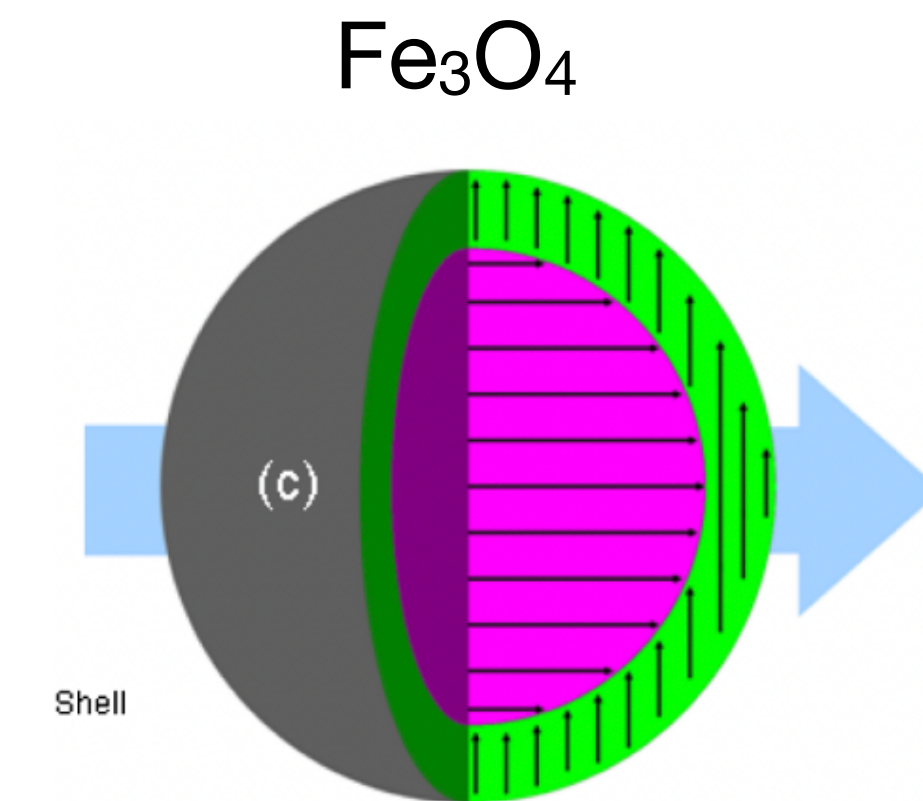
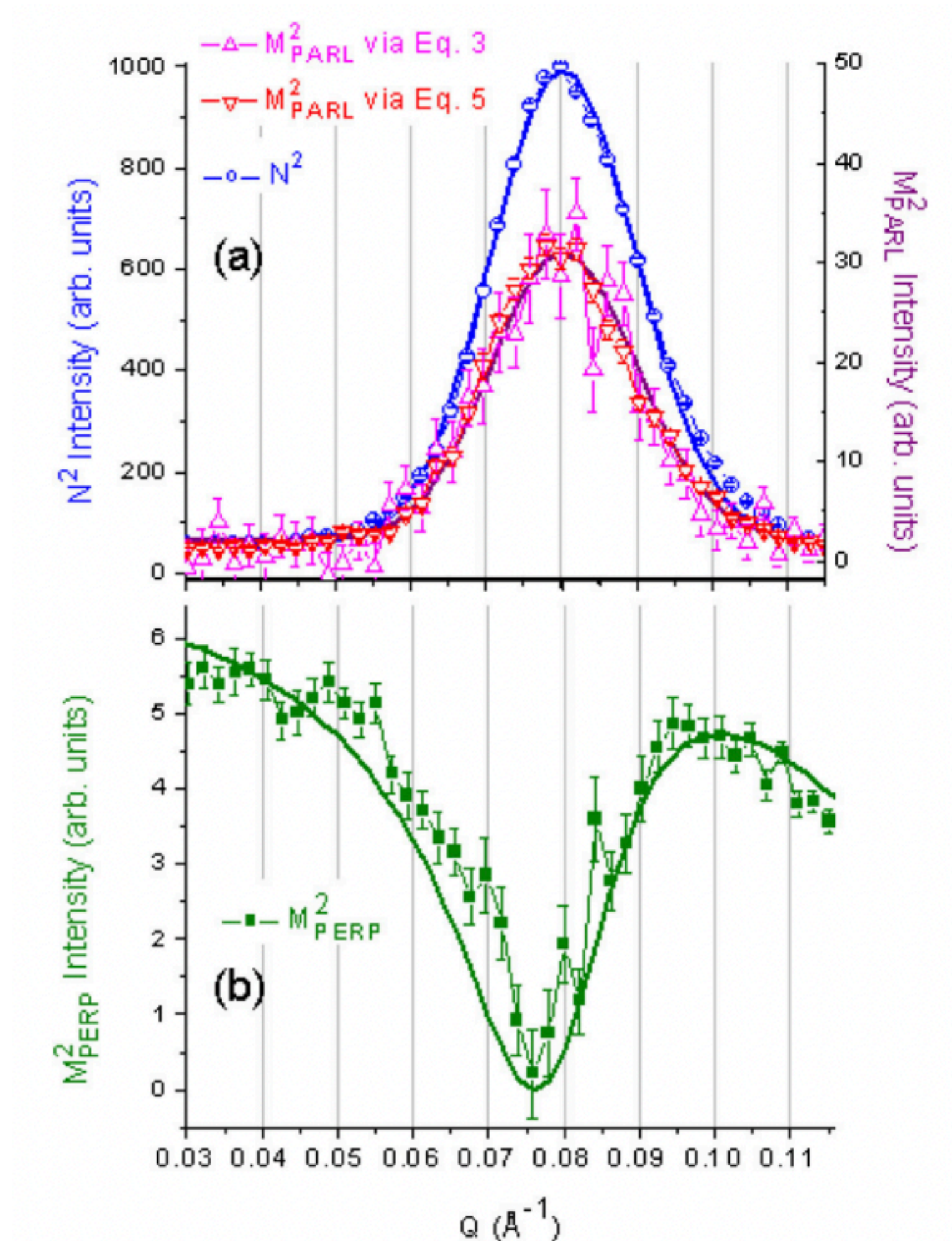
$$M_{\text{PERP}}^2(Q) = \frac{1}{6}(I_{\theta=0^\circ, 90^\circ}^{+-} + I_{\theta=0^\circ, 90^\circ}^{-+}),$$

*Assuming isotropic system with only the X (field) direction being unique

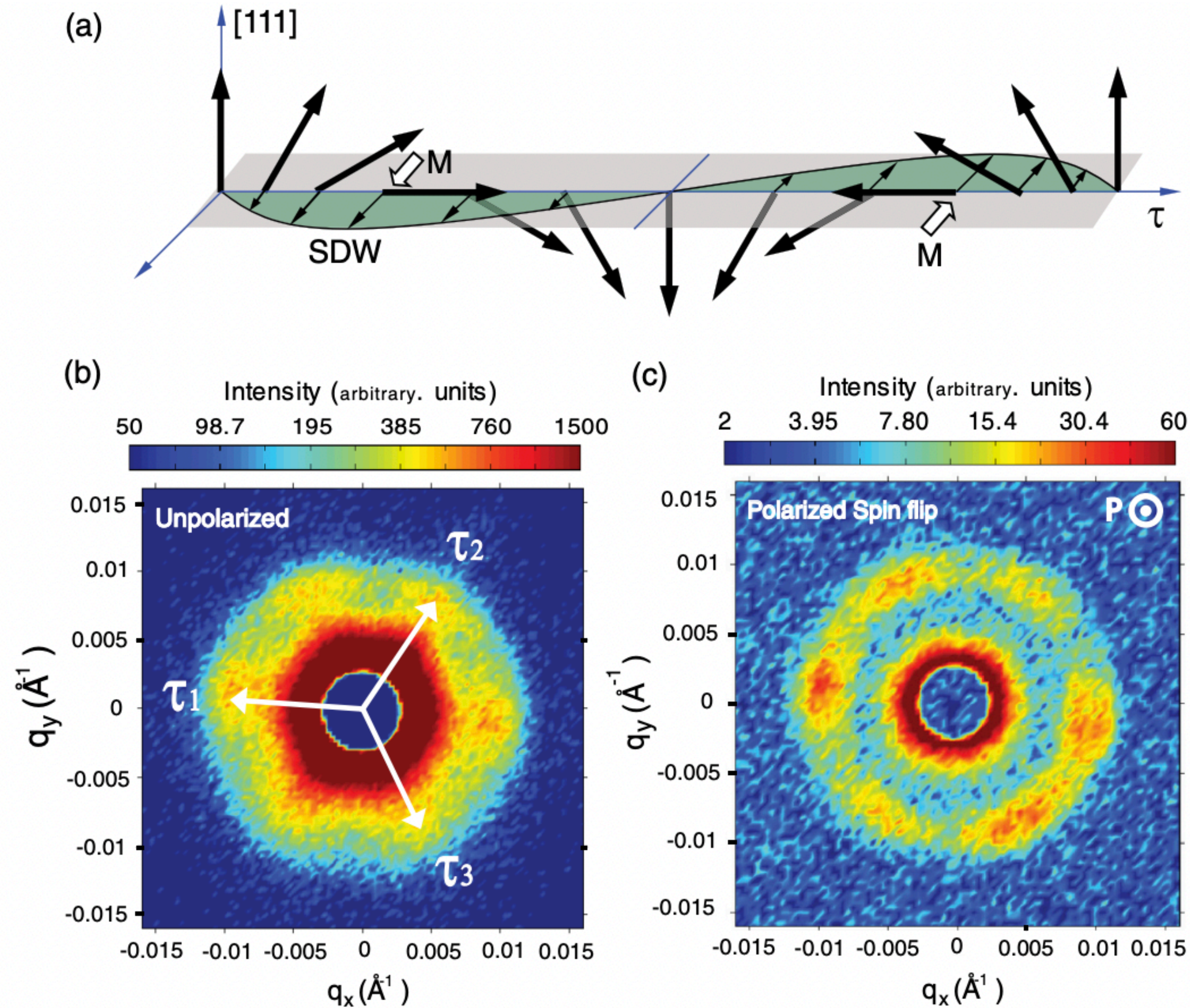
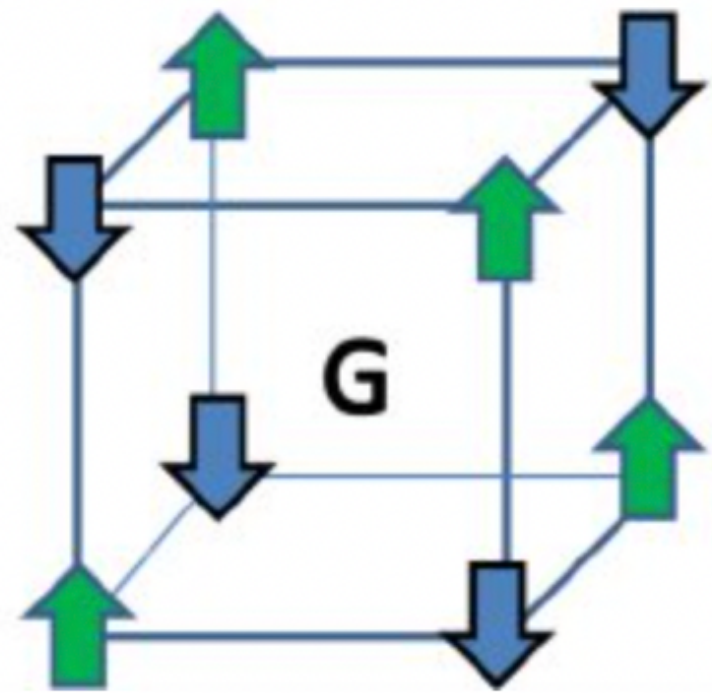
$$M_x^2 = M_{\text{parl}}^2$$

$$M_y^2 = M_z^2 = M_{\text{perp}}^2$$

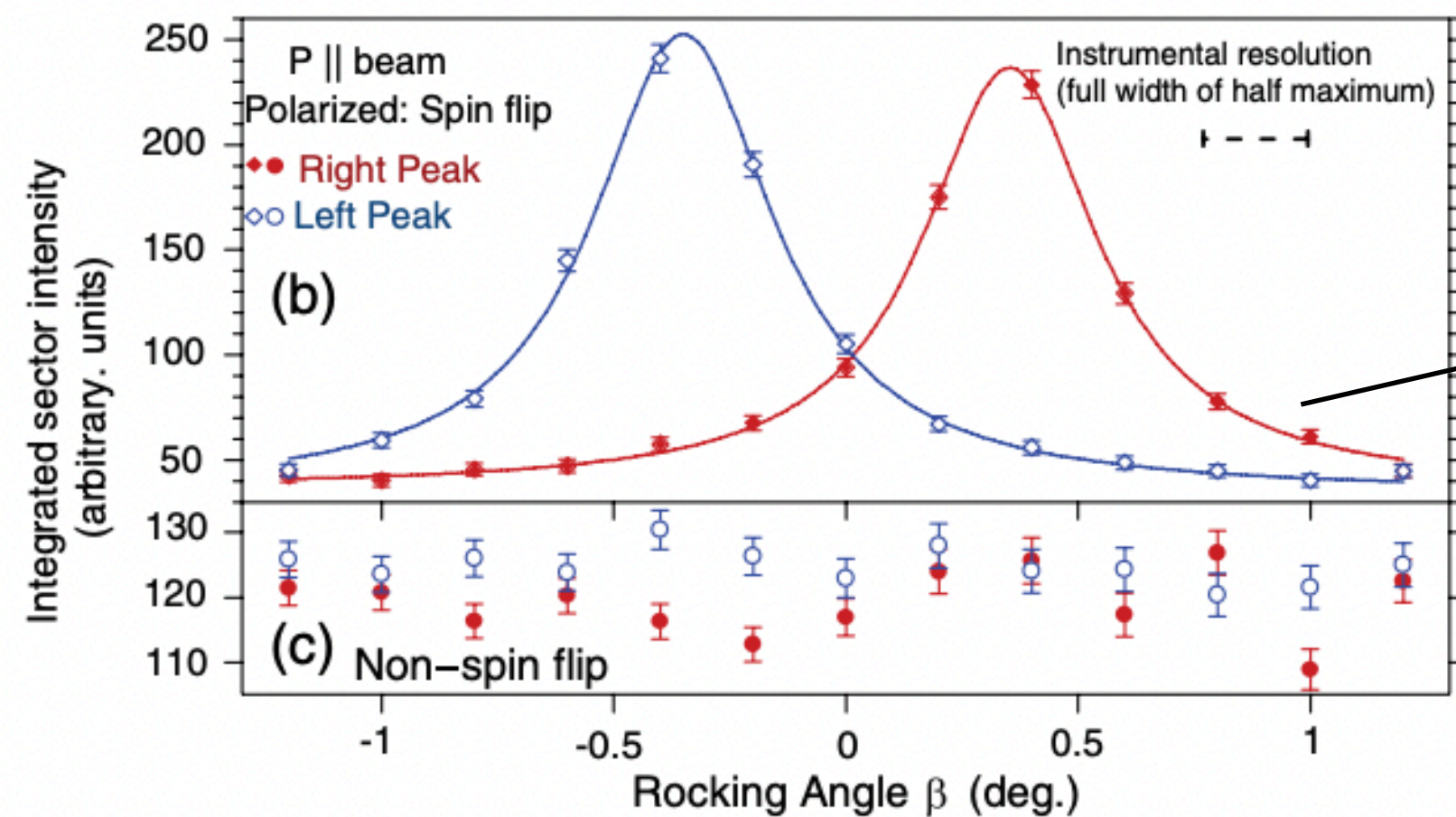
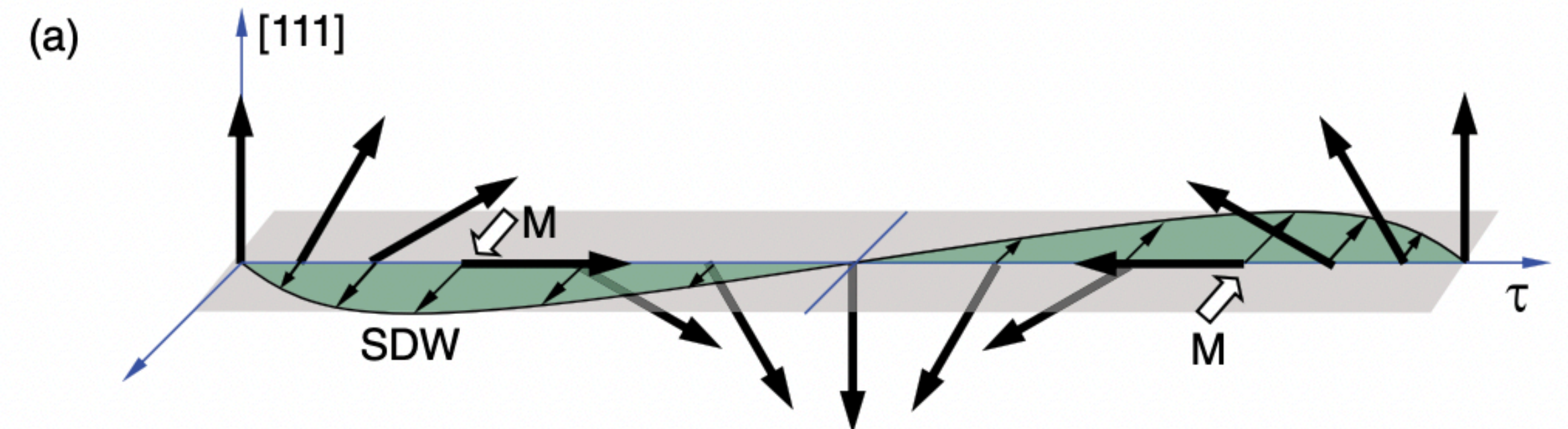
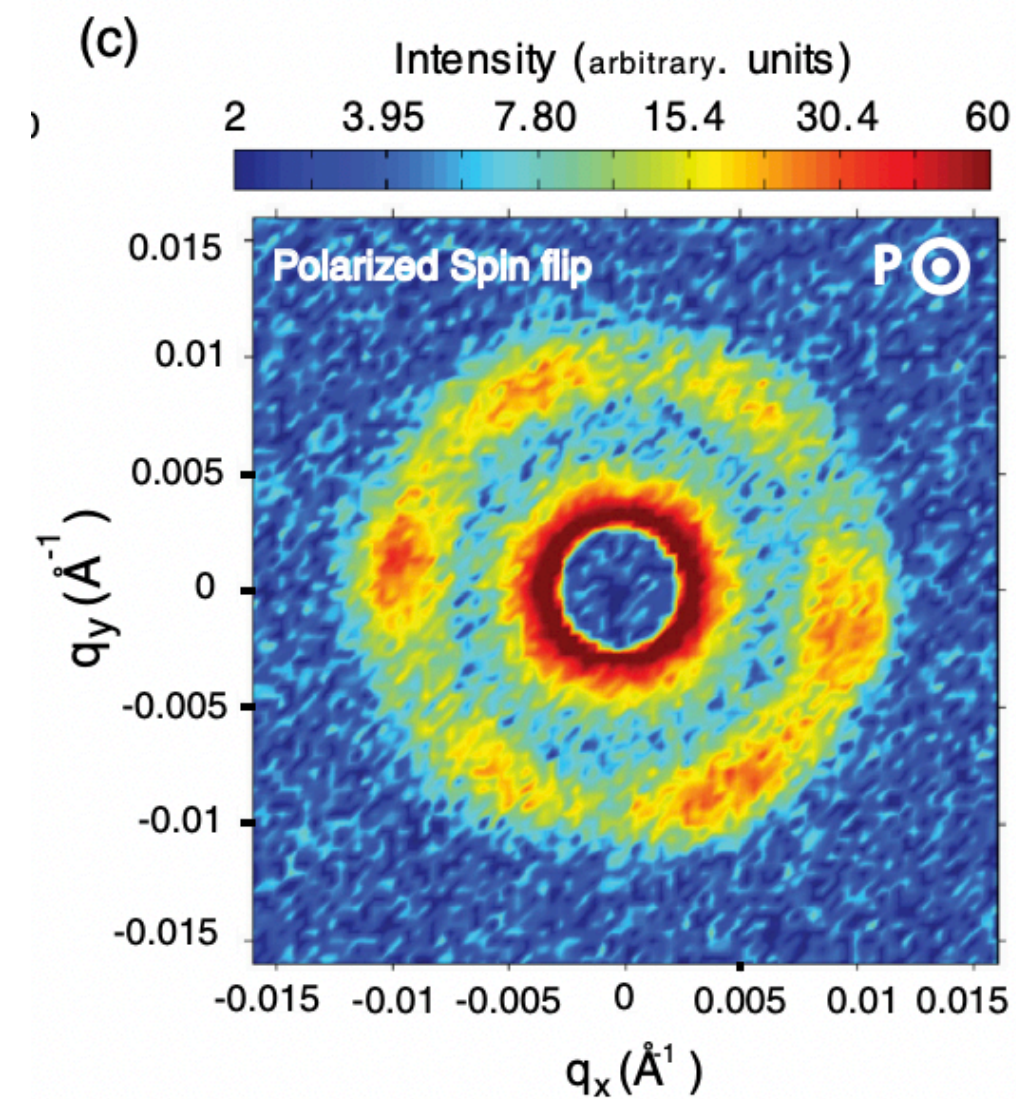
Full-polarized SANS: Characterizing the in-field 3D magnetization profile of NP's



Full-polarized SANS: Characterizing the long-wavelength spin density wave in BiFeO₃



Full-polarized SANS: Characterizing the long-wavelength spin density wave in BiFeO₃

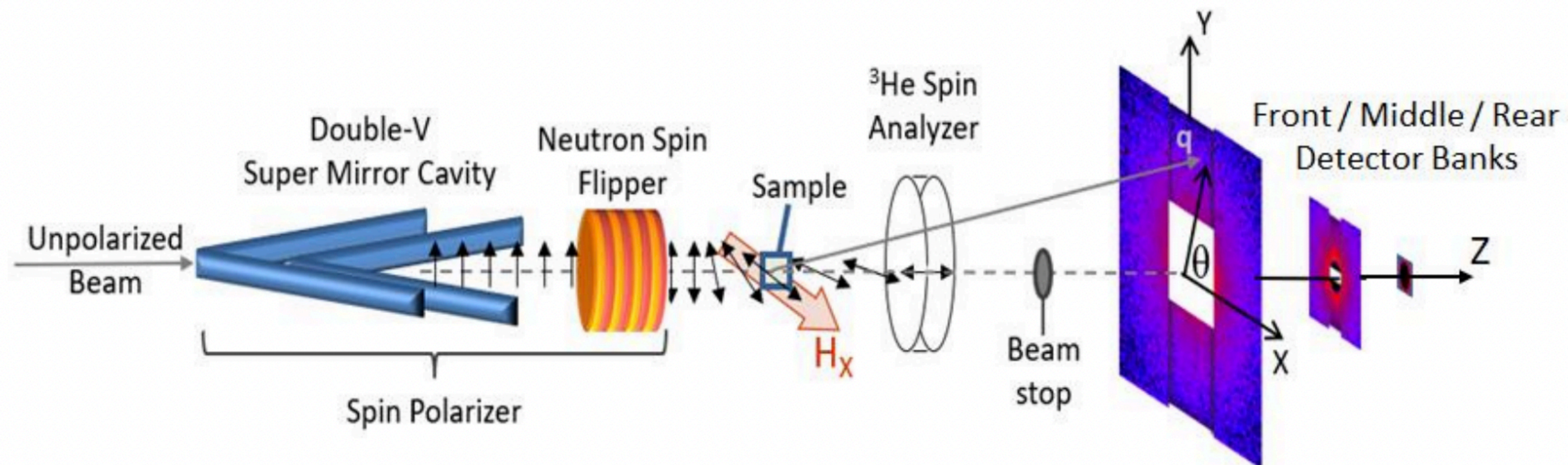


The Bragg peaks only show up in the SF channel for $\mathbf{P}||\text{beam}$.
This implies:

- 1) It is magnetic in origin
- 2) Its spin component is $\perp$ to both $[111]$ and τ

Conclusion

- 1) Magnetic SANS is used to probe magnetic inhomogeneities on a spatial length scale of ~ 1 to $10\,000$ nm.
- 2) NCNR is the host of 5 different SANS instruments including V-SANS, which is a very versatile instrument that also allows for uniaxial polarization analysis.
- 3) Polarized SANS can be used to isolate a weak magnetic signal that lies on top of a huge nuclear scattering signal, to characterize the 3D magnetization profile of NPs, to determine the spin orientation and chirality of spin density wave, and much more...



Conclusion: Please contact us to do polarized SANS at NIST!



Polarized SANS at NIST:

Julie Borchers
John Barker
Wangchun Chen
Shannon Watson

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